Why black holes are difficult

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Penn State, September 10, 2010

Based on:

arXiv:0807.4556  -  JdB, Sheer El-Showk, Ilies Messamah, Dieter van den Bleeken
and work in progress
Outline

1. Higgs $\rightarrow$ Coulomb $\rightarrow$ Geometry
2. Microstates for large supersymmetric black holes
3. Quantum effects in deep throats
4. A bound on the number of smooth supergravity solutions
5. Beyond Geometry?
6. SUSY solutions of $d=3$ supergravity
7. Conclusions
Conventional approach to black holes in string theory:
It would be very interesting could reverse the arrows in this picture and find a geometrical description of the individual degrees of freedom.

Higgs → Coulomb → Gravity

$g_s \to \infty$
Difficult to do in general: look at examples. For 4d black holes in e.g. IIA/CY, relevant gauge theory is N=4 SUSY quantum mechanics (ignoring internal features of wrapped branes and various other subtleties).

Denef; Balasubramanian, Gimon, Levi; Denef, Moore

Simplest case: $U(1) \times U(1)$ gauge theory with N bifundamentals. Field content:

\[ Q_a \]

\[ \vec{X}_1 \rightarrow \vec{X}_2 \]
The Higgs branch of the theory is

\[ \mathcal{M}_H = \{ \sum |Q_a|^2 = \theta \} / U(1) \]

This is equal to \( \mathbb{CP}^{N-1} \). Ground states correspond to cohomology classes and the total number is \( N \).

The Coulomb branch of the theory is a two-sphere

\[ \mathcal{M}_C = |\vec{X}_1 - \vec{X}_2| = \frac{N}{\theta} \]

Ground states are Landau levels due to an effective magnetic field. There are \( N \) of these.
The number of degrees of freedom remains the same! Can see how the two are related by taking $g_s \to \infty$ in the classical theory. Coulomb branch fields become non-dynamical:

$$\vec{X} = \frac{\sum_a \bar{\psi}^a \bar{\sigma}_a \psi_a}{\sum_a |Q_a|^2}.$$  

Act on Higgs branch as the Lefschetz SU(2). Form a finite dimensional representation of SU(2). Commutation relations agree with symplectic structure on Coulomb branch. Form what is commonly referred to as a fuzzy two-sphere.

$$\mathcal{M}_C(\text{fuzzy}) \subset H^*(\mathcal{M}_H).$$
As the string coupling increases, the support of the wave functions moves from the Higgs to the Coulomb branch.
In general, the Coulomb branch is realized as a fuzzy manifold inside the Higgs branch, but does not see the entire Higgs branch. Precise geometric criterion which states disappear is unclear at present.

It appears this typically happens whenever the gauge theory has a “scaling” regime, where on the Coulomb branch one can take some $|\vec{X}_i - \vec{X}_j| \to 0$. More on this later. If there is no scaling regime, in all examples, no states disappear, would be interesting to prove in general.

The next step is to map the Coulomb branch to spacetime geometries. The relevant geometries are known special cases of multi-centered 4d black hole geometries.
Large supersymmetric black holes carrying electric charge $Q$ and magnetic charge $P$ exist in four dimensions. ($P$ and $Q$ can be vectors with many components). cf Bossard/Denef’s talk

There exists however a much larger set of solutions of the gravitational field equations, which includes bound states of black holes, and also many smooth solutions.

Lopes Cardoso, de Wit, Kappeli, Mohaupt; Denef; Bates, Denef; Balasubramanian, Gimon, Levi

Put black holes with charges $\Gamma_i = (P_i, Q_i)$ at locations $\vec{x}_i \in \mathbb{R}^3$
There are corresponding solutions of the field equations only if (necessary, not sufficient)

\[ \langle h, \Gamma_i \rangle + \sum_{j \neq i} \frac{\langle \Gamma_j, \Gamma_i \rangle}{|\vec{x}_j - \vec{x}_i|} = 0 \]

Here, \( \langle \Gamma_1, \Gamma_2 \rangle = P_1 \cdot Q_2 - P_2 \cdot Q_1 \) is the electric-magnetic duality invariant pairing between charge vectors. The constant vector \( h \) determines the asymptotics of the solution.

Solutions are stationary with angular momentum

\[ \vec{J} = \frac{1}{4} \sum_{i \neq j} \langle \Gamma_i, \Gamma_j \rangle \frac{\vec{x}_i - \vec{x}_j}{|\vec{x}_i - \vec{x}_j|} \]
These describe precisely backreacted Coulomb branch solutions. The (one-loop) D-term equations of the gauge theory

\[ \theta_i + \sum_{j \neq i} \frac{N_{ij}}{|x_j - x_i|} = 0 \]

\[ N_{ij} = \# \text{bifundamentals} \]

are the same as the supergravity equation

\[ \langle h, \Gamma_i \rangle + \sum_{j \neq i} \frac{\langle \Gamma_j, \Gamma_i \rangle}{|x_j - x_i|} = 0 \]
For suitable choices of the charges, the solutions do not describe bound states of black holes but smooth solutions.

This happens (thinking about the charges as coming from IIA on a CY) whenever the charges correspond to D6-branes with world-volume gauge fields turned on, or D0-branes. The solutions then become smooth when uplifted to five dimensions.
Only when the supergravity solution is smooth it is honestly geometrical. Singularities and sources require extra input.

These smooth solutions are then candidate microstates for the black hole we started out with (Mathur….) More precisely, they form a classical phase space which after quantization provides the required microscopic degrees of freedom.

Note that it is easy to arrange a situation so that the charges carried by the smooth geometries are identical to those of a large supersymmetric black hole.

To quantize need the symplectic form (or the magnetic field in the Landau level language)
Black hole charge: $\Gamma$

$g_s \to 0$

Higgs branch, $\Gamma = \sum_i \Gamma_i$

Coulomb branch

supergravity

$\vec{X}_3 \Gamma_3$

$\vec{X}_4 \Gamma_4$

$\vec{X}_5 \Gamma_5$

$\vec{X}_1$

$\vec{X}_2$
Set of smooth solutions

Full phase space = set of all solutions of the equations of motion.

\[ \omega \sim \int d\Sigma^{\mu} \left( \delta \frac{\delta L}{\delta (\partial^{\mu} \phi)} \wedge \delta \phi \right) \]
Result:

\[\omega = \frac{1}{4} \sum_{p \neq q} \langle \Gamma_i, \Gamma_j \rangle \epsilon_{i,j,k} (\delta(x_p - x_q)^i \wedge \delta(x_p - x_q)^j) (x_p - x_q)^k |x_p - x_q|^3\]

Can now use various methods to quantize the phase space, e.g. geometric quantization. Can explicitly find wavefunctions for various cases.

In this way, find purely geometric duals for some components of the Coulomb branch.
Of particular interest: **scaling solutions**: solutions where the constituents can approach each other arbitrarily closely.
In space-time, a very deep throat develops, which approximates the geometry outside a black hole ever more closely.

None of these geometries has large curvature: they should all be reliably described by general relativity.

However, this conclusion is incorrect!

The symplectic volume of this set of solutions is finite. Throats that are deeper than a certain critical depth are all part of the same ħ-size cell in phase space: wavefunctions cannot be localized on such geometries.

Quantum effects become highly macroscopic and make the physics of very deep throats nonlocal.

This is an entirely new breakdown of effective field theory.
Wave functions have support on all these geometries
As a further consistency check of this picture, it also resolves an apparent inconsistency that emerges when embedding these geometries in AdS/CFT.

This is related to the fact that very deep throats seem to support a continuum of states as seen by an observer at infinity, while the field theories dual to AdS usually have a gap in the spectrum.

Bena, Wang, Warner

The gap one obtains agrees with the expected gap $1/c$ in the dual field theory (the dual 2d field theory appears after lifting the solutions to five dimensions and taking a decoupling limit).
This non-local breakdown of effective field theory near the horizon is perhaps exactly the sort of thing one needs in order to reconcile the information paradox with effective field theory.

(It has been argued that the information paradox cannot be resolved in perturbation theory)

Notice that the scale that appears is $1/c$, which seems to be a scale that appears often in this context. Evidence for a universal underlying long string picture?

The breakdown is somewhat reminiscent of the breakdown of the statistical description of near-extremal black holes when $\left| T \left( \frac{\partial T}{\partial M} \right) \right| \ll T$.

Preskill, Schwarz, Shapere, Trivedi, Wilczek
What do these smooth geometries actually represent? Do they really represent microstates of the black hole?

Need a cleaner setup: embed in AdS/CFT. The dual CFT has a Hilbert space \( \mathcal{H} = \bigoplus_{\Gamma} \mathcal{H}_{\Gamma} \) and the states corresponding to the black hole span \( \mathcal{H}_{\text{BH}} \subseteq \mathcal{H}_{\Gamma} \). Do the smooth geometries contribute to \( \mathcal{H}_{\text{BH}} \) or to other states in \( \mathcal{H}_{\Gamma} \)?

Non-scaling solutions can be taken apart by changing moduli, while the black hole is still there. These do not contribute to the space of states of the black hole \( \mathcal{H}_{\text{BH}} \).

Scaling solutions on the other hand can not be disentangled from the black hole. In addition, they resemble a black hole (outside the horizon) arbitrarily accurately. They therefore do contribute to the space of states of the black hole.
As has been argued by e.g. by Sen, the full partition function has the following structure

\[ Z = Z_{\text{hair}} + Z_{\text{hair}}Z_{\text{BH}} + Z_{\text{hair}}Z_{\text{BH}}Z_{\text{BH}} + \ldots \]

and one may wonder to which terms the smooth solutions we are talking about contribute. One might be inclined to view the smooth solutions as hair, but we argued that scaling solutions contribute to the black hole? Notice that:

• This is not a sum over Euclidean saddle points, smooth solutions can typically not be Wick rotated.

• Smooth solutions are also not obviously related to one-loop determinants.

• Reminiscent of Farey tail expansion, except there \( Z_{\text{hair}} \) only contains polar states which never coexist with black holes.
For the $1/2$-BPS black holes we considered, there is natural split into “hair” and “black holes”.

This is also suggested by considering split attractor flows.
This is rather peculiar: a macroscopic, classical thermodynamic description of a set of microstates coexists with a macroscopic, classical description of one of its microstates!

Analogy: gas of non-interacting atoms. Spread these evenly in the bottom half of a box and give them all equal upward velocity.
This will look like a classical wave going up and down. It is also one of the configurations that contributes to the partition sum of the gas and coexists with it.

This is a rather atypical setup because the atoms do not interact. In general interactions will destroy this picture. In the black hole case the system is BPS which is somewhat similar to non-interacting.

All this suggests that the smooth scaling solutions are highly **atypical** microstates which have a macroscopic description and coexist with the black hole.

But are they indeed atypical??
Are there sufficiently many smooth supergravity solutions to account for the black hole entropy?

A priori not, we lost some states along the way, and this is not a prediction of AdS/CFT.

Largest set we have been able to find:

Cf Denef, Gaiotto, Strominger, vdBleeken, Yin
In terms of standard 2d CFT quantum numbers we find the following number of states:

$$\left( \frac{3}{16} \zeta(3) L_0^2 \right)^{1/3} \quad L_0 \leq c/6$$

$$\left( \frac{3}{2} c \zeta(3) \left( L_0 - \frac{c}{12} \right) \right)^{1/3} \quad L_0 \geq c/6$$

This is less than the black hole entropy, which scales as

$$S \sim 2\pi \left( \frac{c}{6} L_0 \right)^{1/2}$$
Perhaps we are simply missing many solutions?

Try to find upper bound: count the number of states in a gas of BPS supergravitons. Idea is that all smooth BPS solutions are obtained by taking a superposition of free BPS supergravitons and letting the system backreact. Because of the BPS bound, the energy of the system cannot become be lowered.

After all, classical solutions can be thought of as coherent superpositions of gravitons…
Thus we compute the partition function of a gas of BPS supergravitons; spectrum can be read off from the the KK modes of M-theory on CYxS². Result:

\[ Z = \text{Tr}_{\text{NS,BPS}}(-1)^F q^{L_0} y^{\tilde{L}_0 - 1/2} \]

equals

\[ Z = Z_{\{2,1\}}^{2h_{1,2}} + 2 Z_{\{0,1\}}^{h_{1,1}} - 1 Z_{\{1,0\}}^{h_{1,1}} - 1 Z_{\{-1,2\}} Z_{\{0,2\}} Z_{\{1,1\}} Z_{\{2,1\}} \]

where

\[ Z_{\{s,\tilde{h}_{\text{min}}\}} = \prod_{n \geq 0} \prod_{m \geq 0} (1 - y^{m+\tilde{h}_{\text{min}} - 1/2} q^{n+m+\tilde{h}_{\text{min}} + s}) (-1)^{2s+1} \]
We put $y=1$ and compute the asymptotics of this partition function. Result:

$$S \sim \left( \frac{3}{16} \zeta(3) L_0^2 \right)^{1/3}$$

Clearly backreaction will be important. Difficult to deal with, but can impose one dynamical feature: stringy exclusion principle.

The stringy exclusion principle is related to the fact that the spins of primaries in a level $k$ SU(2) WZW cannot exceed $k/2$. Thus we reinstate $y$ and keep only the terms where the power of $y$ is at most $c/6$. 

Maldacena, Strominger
Now we find precisely the same result as before:

\[
S \sim \left( \frac{3}{16} \zeta(3) L_0^2 \right)^{1/3} \quad L_0 \leq c/6 \\
S \sim \left( \frac{3}{2} c \zeta(3) (L_0 - \frac{c}{12}) \right)^{1/3} \quad L_0 \geq c/6
\]

Strongly suggests supergravity is not sufficient to account for the entropy.

Stringy exclusion principle is visible in classical supergravity (and not so stringy).
In particular, this suggests that all attempts to quantize gravity on its own are futile and will never lead to a consistent unitary theory with black holes.

This is of course perfectly fine: string theory was invented to yield a consistent quantum theory of gravity, so it would have been somewhat disappointing if we could get away with gravity alone.

This statement is also supported by the N=4 case, where one can show that multicentered configurations can never contribute to the index.

Dabholkar, Guica, Murthy, Nampuri
Caveat: Aminneborg, Begtsson, Brill, Holst, Peldan

In $d=3$ there are many solutions which are identical to a black hole outside the horizon but have structure behind it. There may even be enough solutions of this type to account for the black hole entropy (Maloney). Not clear whether these solutions should be viewed as pure states though.

Such solutions cannot obviously be made by throwing gravitons in global AdS.

There may be other solutions with a different topology which cannot be viewed as “small” deformations of AdS.
The above counting was in $d=5$. Can repeat arguments in $d=6$.

$$S \sim (L_0^3)^{1/4} \quad \quad \quad L_0 \lesssim c$$

$$S \sim (c^2 L_0)^{1/4} \quad \quad \quad L_0 \gtrsim c$$

In other dimensions find similar results and never recover growth a la Cardy.
Can we say something about the missing states? They got lost when we passed from the Higgs to the Coulomb branch whenever there are scaling solutions.

Cartoon of Coulomb branch:
The missing states on the Higgs branch all sit in the middle cohomology and have no spacetime angular momentum. If anything, they must sit at the scaling point.

Interestingly, the scaling point has zero symplectic volume but in a sense still represents a large class of solutions. These correspond to tree-like AdS2 solutions.

\[ \Gamma_1 + \Gamma_2 + \Gamma_3 \]

Maldacena, Strominger
One could try to quantize these solutions, but one would need to give them small velocities (adiabatic approximation). Superficially leads to a continuous spectrum of non-BPS states, hard to see how many BPS states can arise from the bottom of this continuum.

Thus, staying purely in supergravity, it seems very difficult to get a complete description of the microscopic degrees of freedom of large black holes.

Is it possible to go beyond supergravity without invoking all of closed string field theory?
Beyond supergravity?

Recall that the F1-D0 system can puff up into a supertube, a D2-brane whose cross section can be an arbitrary curve.

Example of Higgs-Coulomb

One can reproduce the number of F1-D0 bound states by quantizing supertubes.

In a suitable duality frame, they can be described by smooth supergravity solutions.
If we T-dualize the F1-D0 system, many other systems can be shown to puff up into supertubes.

In IIA on $T^6$, with D4 branes wrapping the 6789 and D4 branes wrapping the 4589 directions, the resulting supertube is made of an extended object which one gets by T-dualizing an NS5-brane in two transversal directions: a $5^2_2$-brane.

These exotic objects also appear when U-dualizing conventional branes in three dimensions. Their tensions can involve strange powers of $g_s$ such as $g_s^{-3}$, and strange powers of the radii. Elitzur, Giveon, Kutasov, Rabinovici; Obers, Pioline

Above reasoning suggests such exotic branes may play an important role in understanding microstates. JdB, Shigemori
It is difficult to find explicit solutions of wiggly, supersymmetric non-geometric objects which are not simply U-duals of known things.

Can explicitly construct the metric for the $5^2_2$ supertube.

As one moves through the loop, one picks up a non-geometric twist. Not visible at infinity.
Locally these objects are codimension two. Good warm-up exercise: find all supersymmetric solutions of N=16 supergravity in three dimensions.

Scalar moduli space: $SO(16) \backslash E_8(\mathbb{R}) / E_8(\mathbb{Z})$

Supersymmetric solutions must be of the form $\mathbb{R} \times \Sigma$

On $\Sigma$ there are points $P_i$ where `branes' are located. Around such points the scalars pick up a monodromy $g_i \in E_8(\mathbb{Z})$

There can also be monodromy along non-contractible cycles just as in non-geometric compactifications.
A large number of solutions of the field equations can be found by taking Kähler submanifolds of $E_8(\mathbb{R})/SO(16)$ like $E_6(\mathbb{R})/SO(10) \times SO(2)$ or $SO(p, 2)/SO(p) \times SO(2)$.

One can then take any holomorphic map from $\Sigma$ to the Kähler submanifold.

To get monodromies, need logarithmic singularities at $P_i$.

It is rather difficult to analyze the global constraints on the possible choices of points $P_i$ and monodromies $g_i$.

The simplest case to analyze is to take $SL(2, \mathbb{R})/SO(2)$ as Kähler submanifold. The points $P_i$ and monodromies $g_i$ are exactly as in F-theory and must be such that one can construct a K3 by viewing the map as describing a family of two-tori.
The situation in general is unclear: what geometric structures are being described?

Would be nice to find some sort of integrability condition on the possible choices of points $P_i$ and monodromies $g_i$.
Do non-geometric solutions possible carry enough entropy?

If one only includes T-folds, one can describe solutions using a truncation of string field theory which contains both the massless modes of strings as well as their winding modes.

Such a theory has roughly twice as many fields as supergravity and it is hard to see how that would evade the counting arguments made before.

To really get much more entropy would need a theory which contains many more degrees of freedom. Perhaps including the whole tower of U-dual images of fundamental strings will do the job……
OUTLOOK:

Described progress towards understanding which microscopic degrees of freedom of black holes may be visible in gravity and which ones may not. More work needs to be done. Black holes remain difficult.

Extend breakdown of effective field theory and discussion of quantum effects to generic Schwarzschild black holes: AdS/CFT may allow us to make some progress in this direction.

It appears that supersymmetric black holes cannot be described in terms of gravitational degrees of freedom only, but perhaps non-geometric solutions of gravity may allow one to improve the situation.