The Gauge Connection

The curious ways to see the World of a theoretical physicist

Ashtekar and the BKL conjecture

Research Blogging (http://www.researchblogging.org)

Abhay Ashtekar (http://cgpg.gravity.psu.edu/people/Ashtekar/index.html) is a well-known Indian physicist working at Pennsylvania State University. He has produced a fundamental paper (http://prl.aps.org/abstract/PRL/v57/i18/p2244_1) in general relativity that has been the cornerstone of all (http://marcofrasca.files.wordpress.com/2011/02/ashtekar.jpg)the field of research of loop quantum gravity (http://en.wikipedia.org/wiki/Loop_quantum_gravity). Beyond the possible value that loop quantum gravity may have, we will see in the future, this result of Ashtekar will stand as a fundamental contribution to general relativity. Today on arxiv he, Adam Henderson and David Sloan posted a beautiful paper (http://arxiv.org/abs/1102.3474) where the Ashtekar's approach is used to reformulate the Belinski-Khalatnikov-Lifshitz (BKL) conjecture (http://en.wikipedia.org/wiki/BKL_singularity).

Let me explain why this conjecture is important in general relativity. The question to be answered is the behavior of gravitational fields near singularities. About this, there exist some fundamental theorems due to Roger Penrose (http://en.wikipedia.org/wiki/Roger_Penrose) and Stephen Hawking (http://en.wikipedia.org/wiki/Stephen_Hawking). These theorems just prove that singularities are an unavoidable consequence of the Einstein equations but are not able to state the exact form of the solutions near such singularities. Vladimir Belinski (http://en.wikipedia.org/wiki/Vladimir_Belinski), Isaak Markovich Khalatnikov (http://en.wikipedia.org/wiki/Isaak_Markovich_Khalatnikov) and Evgeny Lifshitz (http://en.wikipedia.org/wiki/Evgeny_Mikhailovich_Lifshitz) put forward a conjecture that gave them the possibility to get the exact analytical behavior of the solutions of the Einstein equations near a singularity: **When a gravitational field is strong enough, as near a singularity, the spatial derivatives in the Einstein equations can be safely neglected and only derivatives with respect**

to time should be retained. With this hypothesis, these authors were able to reduce the Einstein equations to a set of ordinary differential equations, that are generally more treatable, and to draw important conclusions about the gravitational field in these situations. As you may note, *they postulated a gradient expansion in a regime of a strong perturbation!*

Initially, this conjecture met with skepticism. People simply have no reason to believe to it and, apparently, there was no reason why spatial variations in a solution of a non-linear equation with a strong non-linearity should have to be neglected. I had the luck to meet Vladimir Belinski at the University of Rome "La Sapienza". I was there to follow some courses after my Laurea and Vladimir was teaching a general relativity course that I took. (http://marcofrasca.files.wordpress.com/2011/02/belinsky.jpg)The course showed the BKL approach and gravitational solitons (another great contribution of Vladimir to general relativity). Vladimir is also known to have written some parts of the second volume of the books of Landau and Lifshitz on theoretical physics. After the lesson on the BKL approach I talked to him about the fact that I was able to get their results as their approach was just the leading order of a strong coupling expansion. It was on 1992 and I had just obtained the gradient expansion for the Schroedinger equation, also known in literature as the Wigner-Kirkwood expansion, through my approach to strong coupling expansion. The publication of my proof happened just on 2006 (see here (http://arxiv.org/abs/hep-th/0508246)), 14 years after our colloquium.

Back to Ashtekar, Henderson and Sloan's paper, this contribution is relevant for a couple of reasons that go beyond application to quantum gravity. Firstly, they give a short but insightful excursus on the current situation about this conjecture and how computer simulations are showing that it is right (a gradient expansion is a strong coupling expansion!). Secondly, they provide a sound formulation using Ashtekar variables of the Einstein equations that is better suited for its study. In my proof too I use a Hamiltonian formulation but through ADM formalism (http://en.wikipedia.org/wiki/ADM_formalism). These authors have in mind quantum gravity instead and so ADM formalism could not be the best for this aim. In any case, such a different approach could also reveal useful for numerical simulations.

Finally, all this matter is a strong support to my view started with my paper on 1992 (http://pra.aps.org/abstract/PRA/v45/i1/p43_1) on Physical Review A. Since then, I have produced a lot of work with a multitude of applications in almost all areas of physics. I hope that the current trend of confirmations of the goodness of my ideas about perturbation theory will keep on. As a researcher, it is a privilege to be part of this adventure of humankind.

Ashtekar, A. (1986). New Variables for Classical and Quantum Gravity *Physical Review Letters, 57* (18), 2244-2247 DOI: 10.1103/PhysRevLett.57.2244 (http://dx.doi.org/10.1103/PhysRevLett.57.2244)

Abhay Ashtekar, Adam Henderson, & David Sloan (2011). A Hamiltonian Formulation of the BKL Conjecture *arxiv* arXiv: 1102.3474v1 (http://arxiv.org/abs/1102.3474v1)

Marco Frasca (2005). Strong coupling expansion for general relativity *Int.J.Mod.Phys. D15 (2006) 1373-1386* arXiv: hep-th/0508246v3 (http://arxiv.org/abs/hep-th/0508246v3)

Frasca, M. (1992). Strong-field approximation for the Schrödinger equation *Physical Review A, 45* (1), 43-46 DOI: 10.1103/PhysRevA.45.43 (http://dx.doi.org/10.1103/PhysRevA.45.43)