Chandrasekhar's Contributions to General Relativity[1]

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I present a bird's eye view of Professor S. Chandrasekhar's many contributions to general relativity, putting them in the context of other developments and evolution of the field as a whole. The article is addressed to scientists who are not experts in this area.

1. Introduction

I was fortunate to be a graduate student in the relativity group at Chicago, founded by Chandra[2]. Over the years, I came to know him well. He was a source of inspiration for over two decades. Although he passed away some 15 years ago, the knowledge that such inner strength as he had is possible has continued to provide me an overall perspective and guidance to this day. It is therefore an honor and a privilege to contribute to this special issue celebrating the centennial of his birth.

Almost from its inception, general relativity was regarded as a triumph of the human intellect, the most beautiful physical theory ever created. However, for almost fifty years since its discovery, the theory was ignored by most physicists and astronomers. “It appeared", as Max Born put it, “a great work of art, to be enjoyed and admired from a distance." This attitude underwent a profound change in the sixties due to major advances, both theoretical and observational. Since then, the subject has moved toward center-stage both in fundamental physics and in a number of areas of astrophysics. For Chandra's own writings on this subject, see [1-6]. Indeed, by now, issues concerning strong gravitational fields are among the central ones both in the very small – the Planck regime – and the very large-galactic centers, quasars, gravitational waves and the early universe. While a number of people have contributed in important ways to bring about this dramatic shift, in the astrophysical realm Chandra played the decisive role: more than any other single researcher, it was he who brought the beautiful creation of Einstein's to its *natural home*, astronomy. Indeed, even this phrase was coined by Chandra in his address to the International Astronomical Union [2]:

General theory of relativity is a theory of gravitation; and like the Newtonian theory of gravitation which it refines and broadens, its natural home is astronomy.

As Sir Martin Rees succinctly put it, “Chandra probably thought longer and deeper about our universe than anyone since Einstein.”

In this article, I will sketch Chandra's contributions to general relativity and relativistic astrophysics. However, I should emphasize that the article is far from being exhaustive. For this, I will not even apologize; I think it would be nearly impossible for anyone to provide an

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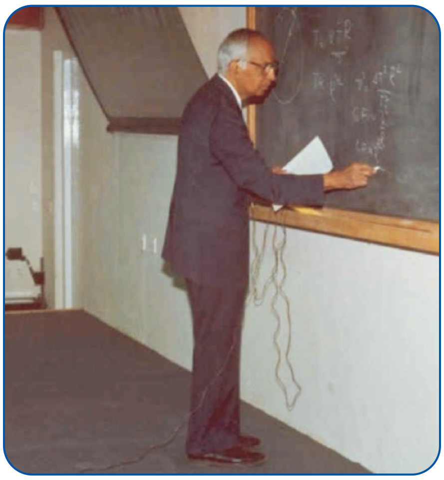
[1] *The article is based on the author's earlier article published in Current Science 70, 800-805 (1996) . Permission from the Indian Academy of Science is acknowledged.*

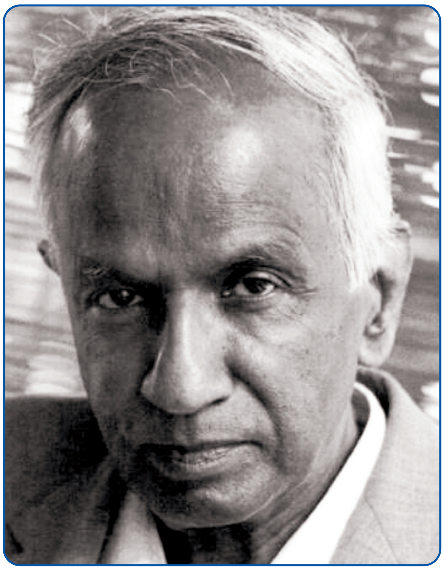
[2] *In this article I will refer to Professor Chandrasekhar simply as Chandra because that is how all of his Friends and colleagues (including graduate students) addressed him.*

exhaustive account of all of Chandra's contributions even to this one area. Also, my article will not be a detailed, technical account, first because the readership of *Physics News* is very diverse, and second, because interested readers will find such summaries in the forewords and introductory chapters of four of the several volumes on Chandra's selected works [7-9]. I will try to make this article complementary to those expositions by putting Chandra's contributions in the context of other developments in general relativity, providing a flavor of the intellectual atmosphere that prevailed during those years. I will also include some anecdotes that, I hope, will shed further light on why certain issues were considered difficult at the time, what the 'mainstream' directions were, and how Chandra influenced the field by opening novel avenues.



Chandra giving a lecture at IUCAA, Pune (Photo courtesy: IUCAA)

Chandra became interested in general relativity in the early sixties. By then he had essentially completed his research on hydromagnetic and hydrodynamic stability and was looking for a new field. His usual way when he changed fields was to think about possibilities, study the literature sufficiently to gain a personal view of the field, perhaps talk to some experts and then come to a decision based on a combination of interest, strengths and temperament. In the case of general relativity, he appears to have taken longer than the usual few months. In 1961, he taught an advanced course on the subject, which, incidentally, was probably the first time general relativity was taught at the University of Chicago [5]! Then, in 1962, he went to Warsaw to attend the third international conference on general relativity and gravitation[3]. This was a most interesting meeting in many ways. As Hawking put it recently in his lecture "My life in Physics", it marked a beginning of the renaissance of general relativity. Established figures like Hermann Bondi, Paul Dirac, Richard Feynman, and Leopold Infeld were all there and so were the 'young Turks' like Ted Newman and Roger Penrose. It was at this meeting that Feynman unveiled his work on the perturbative quantization of general relativity, Penrose explained his new spinorial methods in the classical theory, and Bondi emphasized the reality and the meaning of gravitational waves in full general relativity. The proceedings of this conference are especially illuminating because they contain not only the formal talks but also transcripts of informal discussions. For almost two decades, they continued to serve as a great entry into general relativity and quantum gravity for many students, including me.



During the conference Chandra played the role of an observer. He was impressed by Bondi's lecture and the conference sealed his decision to work in general relativity. However, coming from the background he did, he decided to pursue a direction that was different from the then `mainstream' topics that were discussed and to explore instead the role of general relativity in astronomy. As he explained in his 1978 Oppenheimer memorial lecture [5], Chandra felt that "Einstein's

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[3] *There is an interesting story here. Since Chandra had already accomplished so much in so many different areas, the National Science Foundation (NSF) wanted to support Chandra's travel to Warsaw. But unfortunately, he did not fit in any of the 'standard categories' they then had. "Are you giving an invited talk?" they asked him. "No", he answered. Are you making a presentation?"No", again. "Are you an expert?" "No". He explained that he was in fact going there because he was thinking of entering the field. Finally they gave up, stopped asking these official questions and found the appropriate excuse to provide the travel funds. Considering how much Chandra contributed to general relativity for the subsequent thirty three years, this was probably one the best investments NSF made!*

theory is incredibly rich" but that not enough had been done to extract its physical predictions, i.e., its real content. Indeed, Chandra's lectures and some of the writings of his later years contain remarks on Einstein which may seem critical. It is not that he did not appreciate the depth and aesthetic beauty of Einstein's theory - he emphasized it time and again. Rather, Chandra felt that, in a certain sense, Einstein did not `believe in the theory sufficiently deeply'. For, if he had, he would have tried energetically to work out its physical consequences in the strong field regime where the `soul' of the theory lies. Many of the fascinating results would then have been discovered decades earlier and general relativity would have entered the `mainstream' much sooner. In several of our detailed conversations on the subject, Chandra contrasted Einstein's reluctance to jump into these issues with Newton's immediate grasp and pursuit of the then deep problems in astronomy. This, I believe, is the regret Chandra was expressing when he appears to be critical of Einstein.

2. Stability, Gravitational Waves and Post-Newtonian Approximation

In his very first work on general relativity, therefore, Chandra attacked a problem that had direct astronomical implications. He considered spherical self-gravitating objects and studied the problem of their stability against *radial* perturbations. Thus, one begins with an equilibrium configuration–a static, spherical solution to Einstein's equations with a perfect fluid source– and perturbs it in a spherically symmetric manner. In the first approximation, one linearizes the coupled set of equations around the equilibrium configuration. The key question is whether the perturbations correspond just to small oscillations or if they grow unboundedly in time. In the former case the system is stable, and in the latter, it is unstable. This problem was studied in the Newtonian theory and it was known that if the adiabatic index γ of the perfect fluid source is greater than 4/3 – which is likely to be the case in realistic cases – the system is *stable*. Chandra found that this is no longer the case once general relativity is brought in: *there is a qualitative difference.* In particular, this relativistic instability implies that there is an upper limit on how dense the stars can be. In the hands of Jim Bardeen, Jim Hartle and others, this result grew into the strongest theoretical argument for the existence of black holes with mass a few times that of the sun. It also has important implications for white dwarfs. This work was completed in 1964 and still stands as a landmark in relativistic astrophysics. It is rare indeed for the very first paper in a new area to have such a deep impact on the subject.

Mathematically, the assumption of spherical symmetry made the problem effectively '1-dimensional' and hence a complete solution was possible. However, the assumption is very restrictive from a physical viewpoint. It ruled out rotation of the source and, more importantly, the possibility of emission of gravitational waves. Newtonian theory does not allow gravitational waves; their presence is a purely relativistic effect. And to probe this qualitatively new aspect of general relativity, one must consider non-spherical scenarios in which the quadrupole moment of the body changes in time. Chandra's next series of papers was devoted to the development of a formalism to analyze these issues and especially the question of whether the dissipative effects caused by the back reaction give rise to secular instabilities which, of course, would be absent in the Newtonian theory.

Up until the early sixties, however, this area was riddled with conceptual puzzles and uncertainties. For it was not clear then that full, non-linear general relativity allows gravitational radiation. In the weak field, linearized approximation *around flat space-time*, the problem was solved by Einstein just a year after the discovery of general relativity. In this approximation, there were indeed gravitational waves. However, in the full theory, the issue of disentangling gauge from dynamics is non-trivial and there was some confusion as to whether the waves were purely coordinate effects which could be `transformed away.' In 1922, the celebrated Astronomer Arthur Eddington argued that this was indeed the case and therefore gravitational waves" traveled with the speed of thought!" It is interesting to note that in 1936 Einstein himself wrote to Max Born:

> *Together with a young collaborator [Nathan Rosen], I have arrived at the interesting result that gravitational waves do not exist, though they had been assumed to be a certainty to the first approximation. This shows that the nonlinear field equations can show us more, or rather limit us more, than we have believed up till now.*

The paper was submitted to Physical Review and, it is now known, refereed by Howard P. Robertson, a leading relativist, who wrote a detailed report pointing out that the conclusion was incorrect. A curious incident then followed and the paper was withdrawn from Physical Review. Later in a discussion with Leopold Infeld and Einstein, (apparently without revealing that he was the referee) Robertson pointed out that Einstein's erroneous conclusion was the result of a bad coordinate choice! A revised version of the Einstein-Rosen appeared in the Journal of the Franklin Institute without the “disproof" of the prediction of gravitational radiation.

During the Warsaw conference - Chandra's first foray in general relativity – an entire day was devoted to the issue of reality of gravitational waves in full general relativity[4]. In light of such confusion, it is remarkable that even though the field was so new to him, Chandra quickly zeroed in on the key issues and harbored no doubts on the reality of gravitational waves in full general relativity. But he often explained to students why the issue was not crystal clear at the time. In particular, explicit solutions were known - for example the so-called C-metric discovered by Levi-Civita already in 1918 - which seemed to admit gravitational waves and yet, looked at from a different perspective, seemed to be time independent (i.e. static). (Much later in 1982 Tevian Drey and I analyzed the global structure of this space-time in detail and showed that *does* admit gravitational waves.) Furthermore, people were right in not fully trusting the conclusions of the linearized theory. For example, it is now known that (in a spatially compact context which is of interest in cosmology) there are solutions to the linearized equations which have no analog at all in the full theory. A number of these issues related to gravitational waves were cleared up around the time of the Warsaw conference and by the mid-sixties, thanks to the analysis by Bondi, Rainer Sachs, Penrose and Newman, the reality of gravitational waves was firmly established.

The techniques used in these calculations are mathematically rigorous and elegant but cannot be applied directly to extract answers to the astrophysical questions in which Chandra was interested. What was lacking was a systematic approximation scheme. Chandra undertook the task of developing one and the work was carried out partly in collaboration with his students Yavuz Nutku and Paul Esposito. The scheme is called the “post-Newtonian approximation" and its aim is to develop a perturbation theory by starting with the Newtonian answers and correcting them step by step. The smallness parameter is $(v/c)^2$ where v is the velocity of the gravitating bodies. The only work along these lines available then was that due to Einstein-Infeld and Hoffman, carried out in the late thirties, which had encountered difficulties (i.e. infinities) because the sources were treated as point particles. Chandra could deal with hydrodynamics and also consider rotating objects. To see the effects of gravitational radiation, one has to keep terms up to $(v/c)^5$, or, in the usual terminology, up to the 2.5 post Newtonian order. Chandra carried out the calculations up to this order and discovered a new secular instability caused by the radiation reaction effects. Not only do gravitational waves exist but one could literally see them making a *qualitative* difference!

Chandra completed his work in this area in late seventies. Since then the post-Newtonian formalism has become standard tool in relativistic astrophysics used in the studies of stars, star clusters and even planetary motions. In the hands of Clifford Will and others it has had an unanticipated application to physics as well: it has led to what is called a parameterized post-Newtonian formalism which provides a systematic method to compare and contrast various relativistic theories of gravity. In particular, a large number of alternatives to general relativity could be ruled out by comparing the values of their post-Newtonian parameters to those observed experimentally. More recently, the post-Newtonian methods have also been used to study the `death dance' of coalescing binaries and the gravitational radiation emitted in the process. The resulting wave forms supply critical theoretical inputs for the analysis of signals that one expects to see in the

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[4] *Feynman was disturbed by this and sent a (now public) strongly worded letter to his wife. In retrospect, it appears that since he had strong confidence in perturbative methods, he did not pay due attention to the subtleties associated with the fact that in full general relativity there is no space-time in the background to perturb around; everything including geometry is dynamical. Even today in full general relativity the problem of angular momentum carried away by gravitational waves is not completely resolved because of these conceptual issues.*

LIGO and VIRGO gravitational wave detectors. In recent years, interesting methods from quantum field theory have been introduced to carry out calculations more efficiently! In spite of these and other applications, however, some basic questions still remain. For example, it is now known that the perturbation series is not likely to converge and a well-controlled analysis of error-estimates is still unavailable. It is not even known if the series is asymptotic. It is perhaps because of this open-endedness that, when his research work was complete, Chandra did not, in his usual fashion, write a definitive monograph on this subject. He is said not to have had the 'aesthetic feeling of completeness and coherence' that he had in other areas.

In 1970, Chandra decided that he would, once and for all, study in detail the recent results in general relativity. These involved the so-called "global techniques" and the underlying differential geometric methods were quite different from the more classical mathematical techniques over which Chandra had complete mastery. However, the new methods were essential to analyze such physical issues as the properties of black holes and the nature of gravitational radiation in exact general relativity. In his unique way, Chandra invited three bright young relativists, Brandon Carter, George Ellis and Robert Geroch -all in their late twenties - for a summer in Chicago and had them give lectures. Chandra followed the courses with great diligence, taking notes and calling the lecturers, sometimes late at night, when he had a difficulty. And he was then nearing his 60th birthday! It is this unique spirit that kept Chandra intellectually fresh forever. Soon after that, Bob Geroch and Jim Ipser joined the faculty and a relativity group was created at Chicago. From then on, there were regular relativity seminars and, as a general rule, Chandra attended them all.

However, in his own research, he continued to follow his own inner instincts and the next topic he undertook was motivated by his previous experience with Newtonian gravity: he returned to the analysis of the structure and stability of rotating stars but now in the framework of general relativity. Most of this work was carried out in collaboration with his student John Friedman. They developed a framework which parallels the one for Newtonian gravity (which itself is based largely on some earlier work of Chandra's) and analyzed the issue of stability against small perturbations. Under the assumption of slow rotation, they found a qualitative similarity with Newtonian results: rotation tends to stabilize the star. The problem was also investigated by Hartle, Kip Thorne and Kumar Chitre around the same time. However, while these authors used a computer to do the complex algebra, the Chandrasekhar - Friedman work, of course, was all done by hand. These papers provided a foundation for a systematic analysis of the stability of relativistic rotating stars which culminated, several years later, in Friedman's result that, in general relativity, *all* rotating stars are unstable due to radiation reaction. It is no wonder that Chandra's intuition about the importance of radiation reaction in general relativity is widely regarded as a deep insight in relativistic astrophysics.

I must pause to marvel also at Chandra's impeccable sense of timing. He came the subject just a few years ahead of the dramatic inputs from observations. The subsequent discoveries of quasars and pulsars gave credence to the existence of massive, compact objects in the universe and general relativity suddenly became relevant to astrophysics. Quasars need engines and black-holes are the only plausible candidates. Pulsars are compact, rapidly rotating bodies and hence the issue of relativistic stability became critical. In the early seventies, Russell Hulse and Joe Taylor discovered the first binary pulsar and the effect of radiation reaction on the orbits of such bodies now had direct observational consequences. If one had a crystal ball to foresee these developments before choosing research problems, one could not have made better choices than the ones Chandra's inner voice led him to.

3. Black Holes & Colliding Waves

The next major topic Chandra took up was black-holes. He was fascinated by the elegance and simplicity of the underlying theory. He began his treatise, *Mathematical Theory of Black Holes* by summarizing why:

> *Black holes are the most perfect macroscopic objects there are in the universe: the only elements in their construction are our concepts of space and time.*

From mid seventies to late eighties he wrote a number of influential papers in this area. This work brings out a number of his unique traits. First, Chandra always liked to develop his own perspective of the subject, his own ways of tackling problems. Second, he disliked loose ends and incompleteness. Therefore, he decided to re-examine perturbations of the Schwarzschild black-hole and brought harmony and coherence to the subject by explicitly showing that the various equations that had emerged in different approaches were in fact equivalent. While most of us would be satisfied with this accomplishment, Chandra did not stop here. He deeply believed looking at the problem from different angles, understanding all its ramifications and bringing out the `wholeness' of the subject. (Indeed, he was often critical of astrophysicists who lacked this broader perspective and were interested in solving only the immediate problem in a `narrow' way.) As a result, he had the leisure to discover an amazing fact. The equations governing the perturbations of this black - hole can be written as (time independent) Schrödinger equations in *two different* ways, i.e., with two entirely different potentials. Yet, because they deal with the same physical problem, all the transmission and reflection coefficients are the same. For this to happen, it turns out, the two potentials have to satisfy an *infinite* number of identities. Therefore, in general, it is very hard to construct such pairs of potentials explicitly. The mathematical theory of black-holes provides such a pair naturally! Chandra often expressed the deep satisfaction at this result.

Chandra's work on black-holes also brings out his deep intuition. From his experience with Newtonian gravity, he came to the conclusion that the quasi-normal modes were a key to the understanding of black-hole perturbations (now for the general rotating case, i.e., not necessarily Schwarzschild). At the time, the problem was being looked at entirely differently, using Green's functions. Subsequent years have shown that not only did Chandra's insight simplify the problem mathematically but also astro-physically. It has turned out that even in the violent collisions of black holes or coalescence of inspiralling binary neutron stars, the final stages of dynamics are well described by a quasi-normal ringing of the single, final black hole. Chandra's early recognition of the quasi-normal modes stands as another landmark the field.

However, from a personal point of view, Chandra derived the greatest satisfaction from another, more mathematical result. While studying perturbations of the Kerr black-hole, his desire for completeness and coherence drove him to study the Dirac equation on the Kerr space-time. He managed to separate the equation, reducing the 4-dimensional problem to 1-dimensional ones. This was the sort of the problem that best suited his temperament. No 'standard' method was available. Intuition was called for. One had to speak to the equation in its own intrinsic language and gently coax it to yield. And the final solution was elegant and the derivation simple. An ideal problem for a classical mathematical physicist! It is no wonder that this effort of Chandra's was appreciated more by applied mathematicians than by hard core astrophysicists. In the subsequent years, it has led to some 50 papers and several theorems in the subject of partial differential equations.

Chandra was nearly seventy five when he completed bulk of his work on black holes. Most scientists would have then retired from active research. But Chandra went on to his next topic, which was to be the last one within general relativity: colliding gravitational waves. In a certain sense, this work is a natural extension of his research on black holes. However, it does have quite a different flavor. In this case, Chandra did not begin with specific physical problems or situations of direct astrophysical interest. Rather, he wanted to explore the wondrous mathematical landscape of general relativity, especially the mysterious tunnels that connect different aspects of the theory. This was also the first time he collaborated extensively with younger colleagues - Basilis Xanthopoulos and Valeria Ferrari - who were *not* his students. With them, Chandra first discovered that there is an underlying similarity and unity between the mathematical theory of black-holes and of colliding waves, the extent of which no one had anticipated. (Chandra has commented on this point in detail in [6].) And then they used this fact very effectively to discover a variety of new radiating solutions of Einstein's equations. Given Chandra's record, one day we may realize that even these aesthetically inspired mathematical results have a deep physical significance as well.

4. Epilogue

There is no doubt that Chandra was enchanted by general relativity. He worked in this area for over thirty years, nearly half of his career.

Quoting Sir Francis Bacon, “There is no excellent beauty that hath not some strangeness in proportion," he gave several examples in which this sublime feature emerges from the theory. In his article *Beauty and the Quest for Beauty in Science* [10], he summed up his admiration as follows:

> *When Henry Moore visited the University of Chicago some ten years ago, I had the occasion to ask him how one should view sculpture: from afar or from nearby. Moore's response was that the greatest sculpture can be viewed - indeed should be viewed - from all distances since new aspect of beauty will be revealed at every scale. Moore cited sculptures of Michelangelo as examples. In the same way, the general theory of relativity reveals strangeness in the proportion at any level in which one may explore its consequences.*

While he worked on a large variety of problems, there was an underlying pattern in his work. He chose problems that were well-formulated and, with the exception of his work on colliding waves, were motivated by central physical issues. He strove to extract the true content of general relativity. Invariably, he wanted to find exact solutions to his well posed problems. And he succeeded in this extremely ambitious task because he had a profound intuition for equations of mathematical physics, unmatched among his contemporaries. He sought coherence, completeness and simplicity. He did not always succeed. The Kerr-Newman perturbations resisted him and hundreds of pages of calculations on properties of these black-holes could not be simplified to his satisfaction. However, these exceptions were rare. He almost always met his goals and by doing so made the subject more comprehensible to the rest of us.

In a 1946 lecture [11] entitled *The Scientist* in a *Works of the Mind* series, Chandra put forward his vision of Science and Scientists. In this lecture, and indeed as he continued to reflect on these matters in later years, he focused only on `Pure Science' but divided it into *Basic* and *Derived*. In more informal conversations with me, he often called scientists in the first domain *theory makers*, and in the second, *problem solvers*. He firmly placed himself in the second camp. And indeed, he was a Master Problem Solver! He did not invent a theory of gravitation. He was not the one who crystallized the precise notion of black holes, nor of gravitational waves in exact general relativity. He did not think in modern geometric terms (although he admired others – such as Roger Penrose – who excel at this). Rather, his arsenal of tools came from the late 19th and early 20th century `classical mathematics'. He had a way with real analysis and differential equations. Even the stubborn ones which had remained opaque to other leading figures seemed to readily open their secrets to him. And he had an unmatched intuition for the astrophysical world. He brought all these superb strengths to our field and made Einstein's world more concrete, more vivid, and more transparent. His quest during these `general relativity years' is perhaps best summarized in the passage from Virginia Woolf's *The Waves* that he himself liked to quote:

> *There is a square; there is an oblong. The players take the square and place it upon the oblong. They place it very accurately; they make a perfect dwelling-place. Very little is left outside. The structure is now visible; what is inchoate is here stated; we are not so various or so mean; we have made oblongs and stood them upon squares. This is our triumph; this is our consolation.*

Acknowledgments:

I would like to thank John Friedman and Kamesh Wali for discussions. This work was supported in part by the NSF grant PHY0854743 and the Eberly research funds of Penn State. This article is based in large part on an essay that appeared in *Current Science* 70, 800-805 (1996).

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