The role of cosmological constant in LQC

LOOP QUANTUM COSMOLOGY WORKSHOP

IGC, 23-25.10.2008

Tomasz Pawlowski [IEM (CSIC), Madrid, Spain]

work by:

A. Ashtekar, TP

W. Kaminski, J. Lewandowski, TP

E. Bentivegna, TP

Purpose of the talk

- LQC: an application of Loop Quantum Gravity methods to symmetry reduced (minisuperspace) models.
- Isotropic flat universe with massless scalar field in LQC:
  Results for $\Lambda = 0$ **(A Ashtekar, P Singh, TP gr-qc/0607039)**:
  change of dynamics due to quantum geometric effects.
  - Existence of large semiclassical (contracting) universe preceding expanding one.
  - Bounce at energy density $\rho = \rho_c \approx 0.82\rho_{\text{Pl}}$.
- Presented work: Inclusion of the cosmological constant
  (preliminary studies in gr-qc/0607039).
  - Questions:
    - Are the qualtitative properties similar ?
    - Is bounce still characterized by ρ_c ?
    - What new properties the models with Λ possess ?
  - Due to distinct mathematical properties of an evolution operator $+$ve and $-$ve Λ have to be investigated separately.

LQC quantization scheme

Considered model: flat isotropic (FRW) universe
Matter content: massless scalar field

- Basic variables:
  geometry: A^i_a, E^a_i in isotropic situation reexpressed in terms of coefficients c, p.
  (Gauss and Diffeomprophism constraints automatically satisfied.)
  matter: field ϕ and conjugate momentum p_ϕ.
- Quantization method following LQG:
  - Geometric DOF: triads p and holonomies h raised to operators.
    Matter DOF: standard (Schrodinger) quantization.
  - Kinematical Hilbert space:
    $\mathcal{H}_{\rm kin} = \mathcal{H}_g \otimes \mathcal{H}_\phi =: L^2(\bar{\mathbb{R}}_{\rm Bohr}, {\rm d}\mu_{\rm Bohr}) \otimes {\rm L}^2(\mathbb{R}, {\rm d}\phi)$
  - Basis of $\mathcal{H}_g$: eigenstates of $\hat{p}$ for convenience labeled by v s.t.
    $\hat{p}\,|v\rangle = 2 \cdot 3^{\frac{1}{6}} \pi\gamma\,{\rm sgn}(v)|v|^{\frac{2}{3}}\,|v\rangle$
- Quantization of Hamiltonian constraint $C_{\rm grav} + C_{\rm matt} = 0$: Its geometric components reexpressed in terms of holonomies (Thiemann method), next raised to operators.

Evolution operator

- Quantum constraint similar to Klein-Gordon equation:

$$\partial_\phi^2 \Psi(v,\phi) = -\Theta\Psi(v,\phi)$$

  - Θ is a difference operator
    $\Theta\Psi(v,\phi) = C^+(v)\Psi(v+4,\phi) + C^o(v)\Psi(v,\phi) + C^-(v)\Psi(v-4,\phi)$,
  - Λ enters C^o only, approximately acts like v^2 potential,
  - Θ is symmetric on the domain $\mathcal{D}$ of finite combinations of $|v\rangle$.
- System reinterpreted as free one evolving with respect to ϕ.
- Few important details:
  - No C-symmetry violation interactions $\Rightarrow$ states symmetric with respect to reflection Π in v.
  - Superselection: Domain of v naturally splits onto family of sets preserved by action of Θ and Π:

$$\mathcal{L}_\epsilon := \{v \in \mathbb{R} : v = \pm\epsilon + 4n, n \in \mathbb{Z}\}.$$

    In consequence $\mathcal{H}_g = \oplus\mathcal{H}_\epsilon$, where $\mathcal{H}_\epsilon$ contains functions supported on $\mathcal{L}_\epsilon$ only.

Observables

- Left-hand side negatively definite, thus we take only positive part of Θ. Two sectors: positive and negative frequency. We take the positive part:

$$-i\partial_\phi \Psi(v,\phi) = \sqrt{|\Theta|}\Psi(v,\phi)$$

- Dirac observables:
  - scalar field momentum:

$$\hat{p}_\phi \Psi(v,\phi) = -i\hbar\partial_\phi \Psi(v,\phi)$$

  - volume at given ϕ:

$$|\hat{v}|_\phi \Psi(v,\phi') = \exp[i\sqrt{|\Theta|}(\phi' - \phi)]|v|\Psi(v,\phi)$$

  - scalar field energy density at given ϕ:

$$\hat{\rho}_\phi = \frac{1}{2}\widehat{V^{-1}|_\phi}\ \hat{p}_\phi^2\ \widehat{V^{-1}|_\phi}$$

    where $\widehat{V^{-1}|_\phi}$ defined analogously to $|\hat{v}|_\phi$.

$\Lambda < 0$

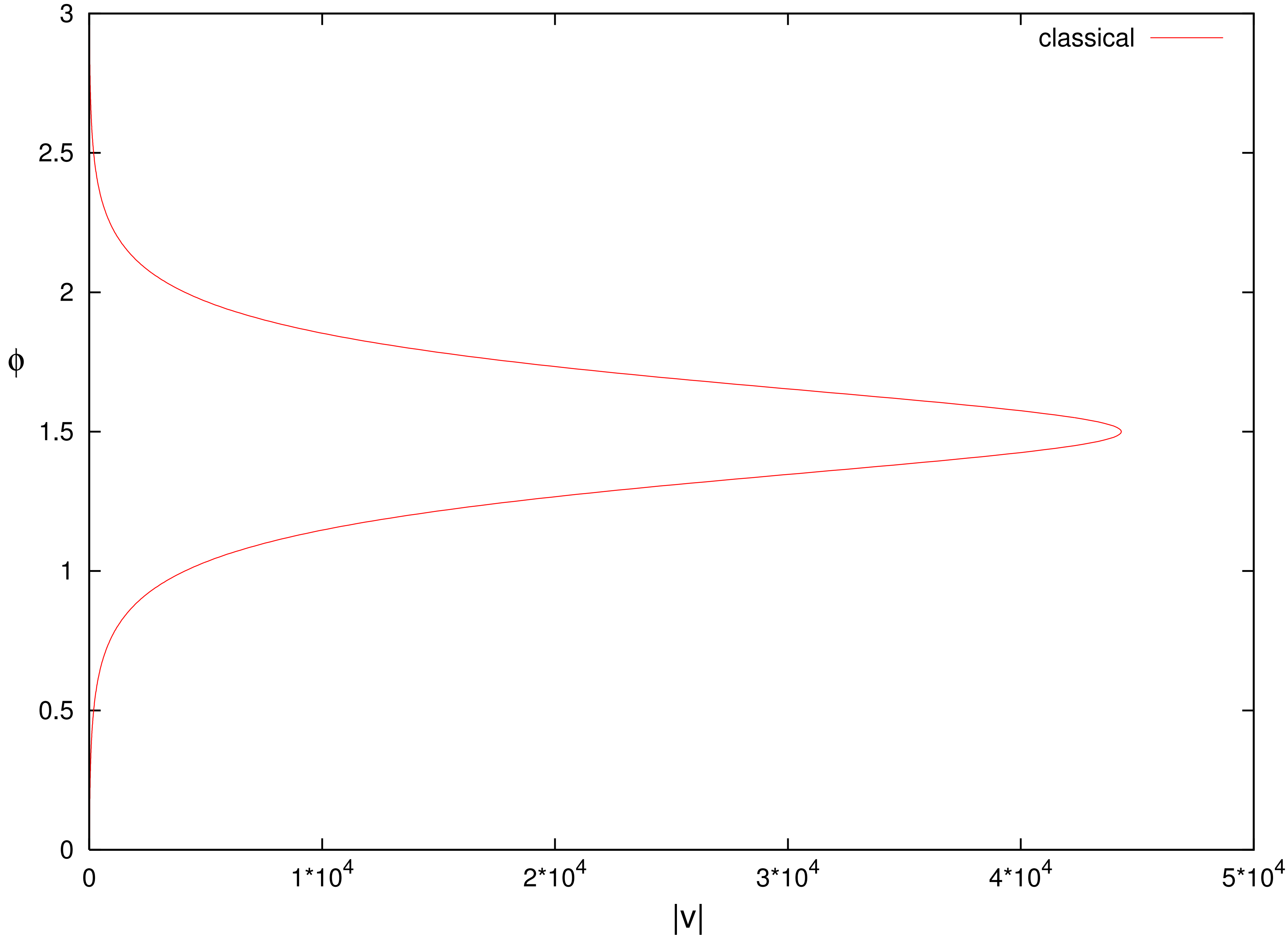
Work by: E Bentivegna, TP

- Classically recollapsing system: recollapse when energy density of ϕ satisfies: $\rho_\phi + \Lambda/8\pi G = 0$.
- Λ acts approximately like $+$ve v^2 potential. Θ is positively definite, essentially self-adjoint, its spectrum is discrete (Lewandowski, Kaminski, Szulc arXiv:0709.3120).
- Normalizable eigenfunctions singled out numerically. Each normalizable eigenspace 1-dimensional. Basis $e_n(v)$ of physical Hilbert space selected out of normalized eigenfunctions.
- Physical states: $\Psi(v, \phi) = \sum_n \tilde{\Psi}_n e_n(v) \exp[i\omega_n(\phi - \phi_o)]$.
- Choice: Gaussian states sharply peaked about $\omega^\star = \hbar^{-1} p_\phi^\star$ and some large $v^\star$ for some initial $\phi = \phi_o$

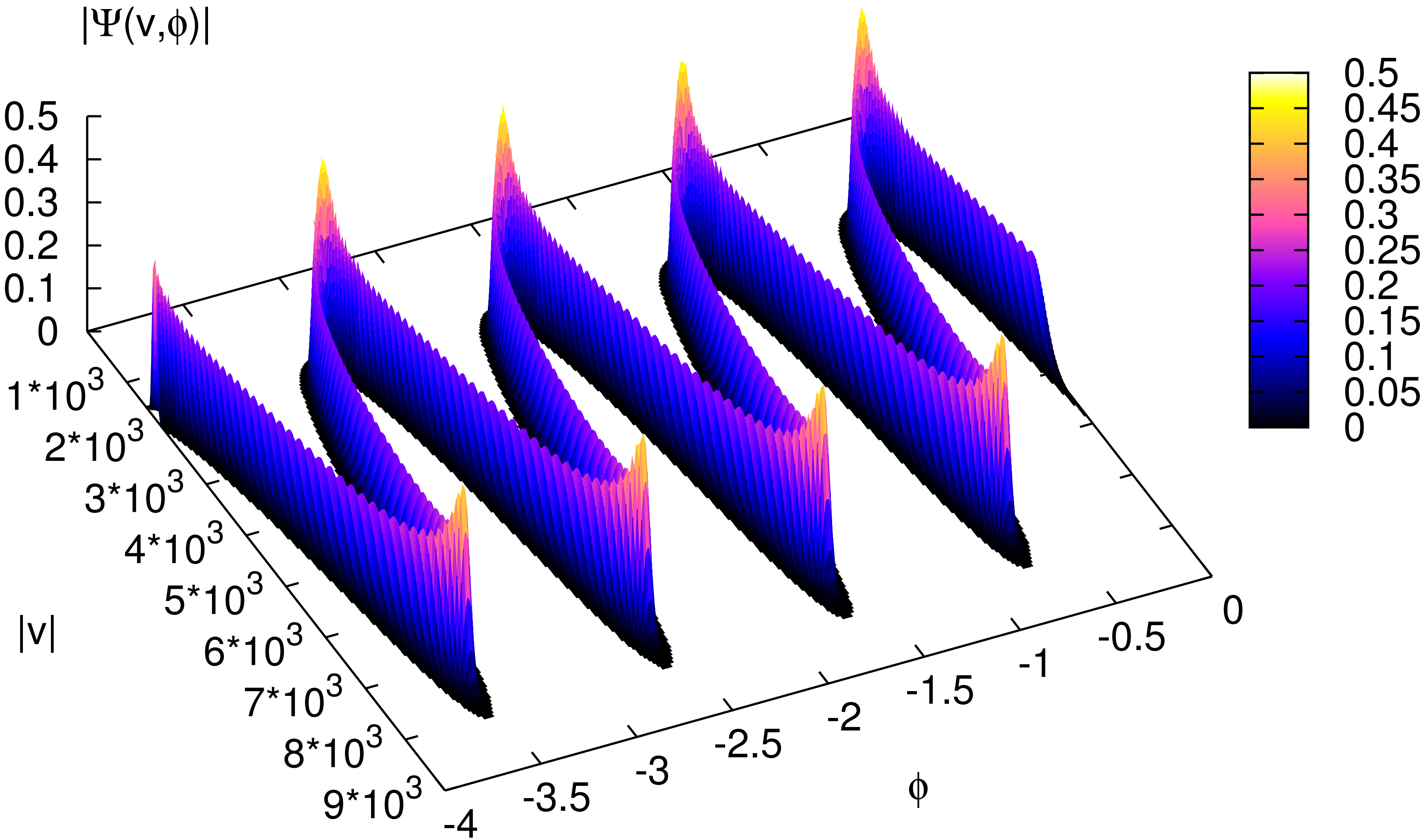
$$\tilde{\Psi}_n = \exp(-(\omega_n - \omega^\star)^2/2\sigma^2)$$

- Dirac observables: $\hat{p}_\phi$, $|\hat{v}|_\phi$, $\hat{\rho}_\phi$.

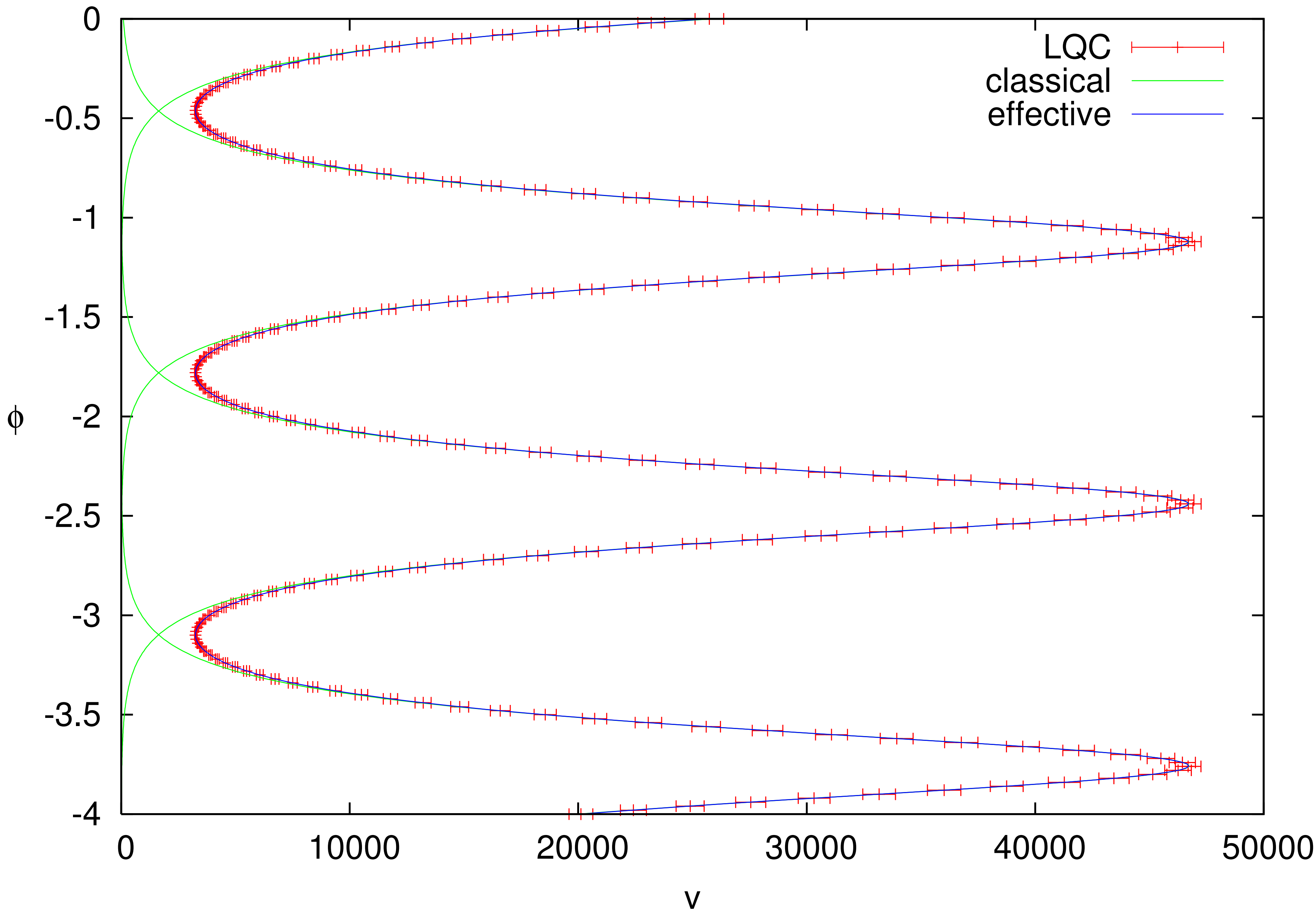
$\Lambda < 0$: *classical trajectory*



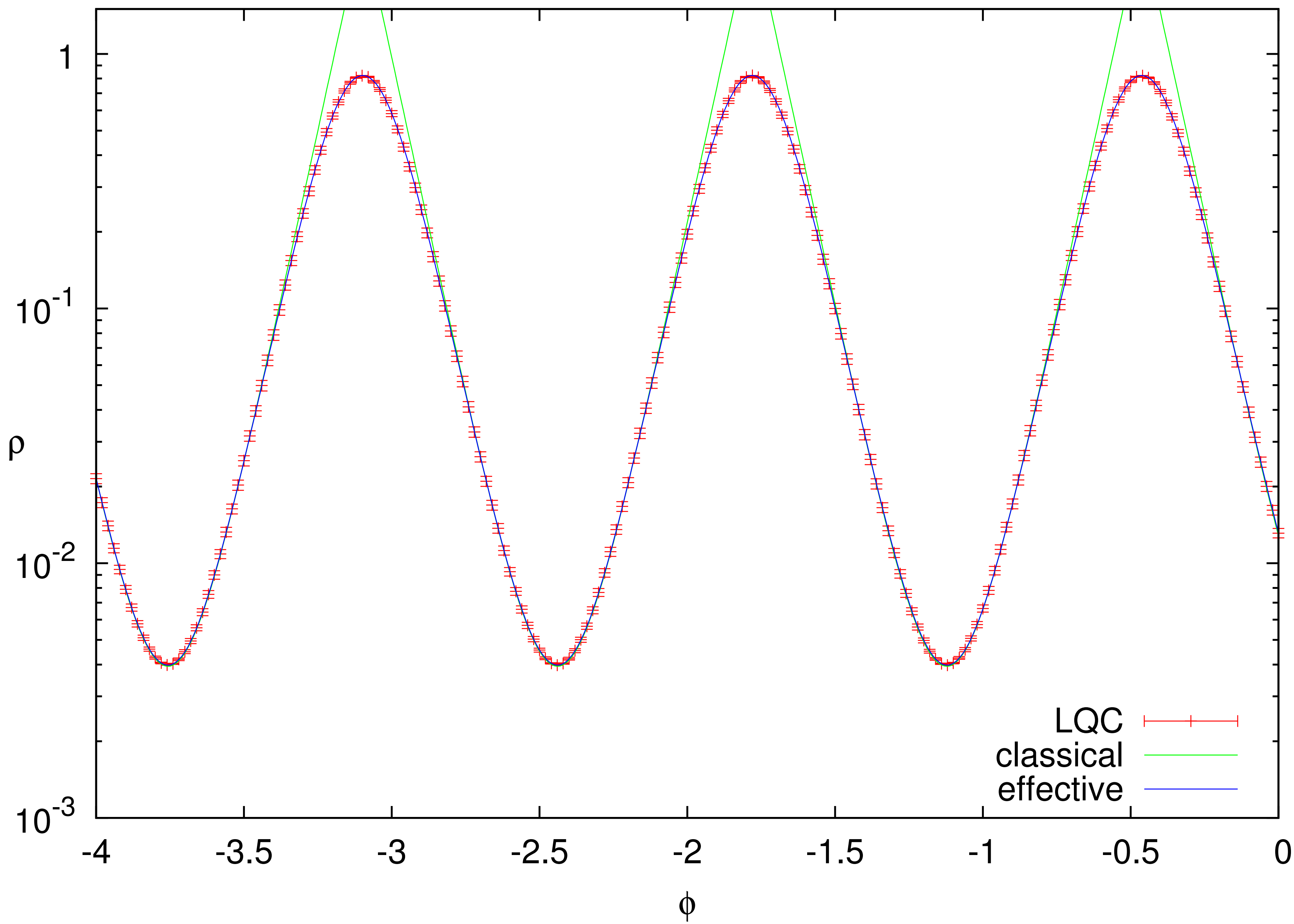
$\Lambda < 0$: *wave function*



$\Lambda < 0$: *quantum trajectory*



$\Lambda < 0$: *energy density*



$\Lambda < 0$ – *results*

- State remains sharply peaked throughout the evolution.
- Expectation values follow classical trajectory till (total) energy density becomes comparable to ρ_c. In particular recollapse at the point predicted by classical theory.
- Bounce exactly at $\rho_\phi + \Lambda/8\pi G = \rho_c$ joins two large semiclassical sectors.
- Singularities are resolved - replaced by a quantum bounce.
- Resulting evolution is periodic (with period depending on Λ).
- Dispersion between cycles:
  - Separation between eigenvalues approaches constant

$$\omega_n - \omega_{n-1} = \Delta\omega(\Lambda) + O(\omega_n^{-2}) \qquad O(\omega_n^{-2}) \leq \frac{A(\Delta\omega)^2}{\omega_n^2}$$

  - Heuristic (numerically confirmed) limit on spread

$$\delta\frac{\Delta v}{v} \leq \tilde{A}\frac{1}{\omega^2}\frac{\delta\omega}{\omega}$$

  - Consequence: very slow spread. For $\Lambda \approx -10^{-120}$ and $V_{\mathrm{MAX}} \approx 1\mathrm{MPc}^3$ dispersion grows twice in 10^{70} cycles.

$\Lambda > 0$

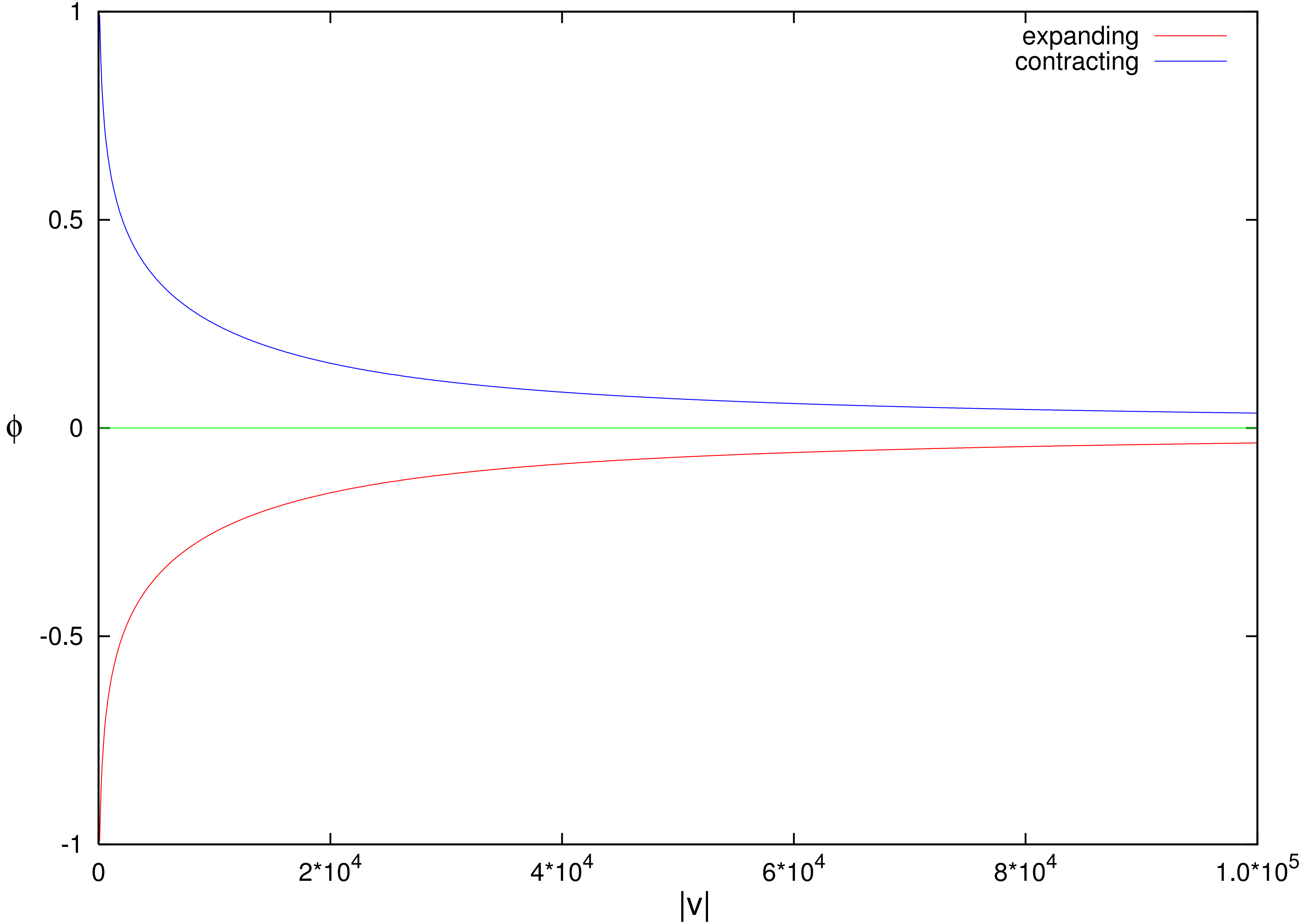
Work by: A Ashtekar, TP

some mathematical aspects: W Kaminski, TP

- Classically two distinct classes: ever-expanding and ever-contracting. In both classes v reaches infinity for finite $\phi = \phi_o$. Solutions parametrized by proper time end there. However ...
  - Original domain of v can be compactified.
  - Classical EOM can be analytically extended. Each solution extends uniquely through $v = \pm\infty$.
  - Behavior of energy density $\rho(\phi)$ (also analytic) shows that procedure doesn't add any new regions but identifies $v = +\infty$ with $v = -\infty$.

  Extended solutions: at infinity universe transits from expanding to contracting phase.
- On the quantum level: contribution from cosmological constant acts approximately as $\propto -v^2$ potential (unbounded from below). Hamiltonians of such system are usually not (essentially) self-adjoint. To verify self-adjointness we analyse the deficiency subspaces.

$\Lambda > 0$: *classical trajectory*

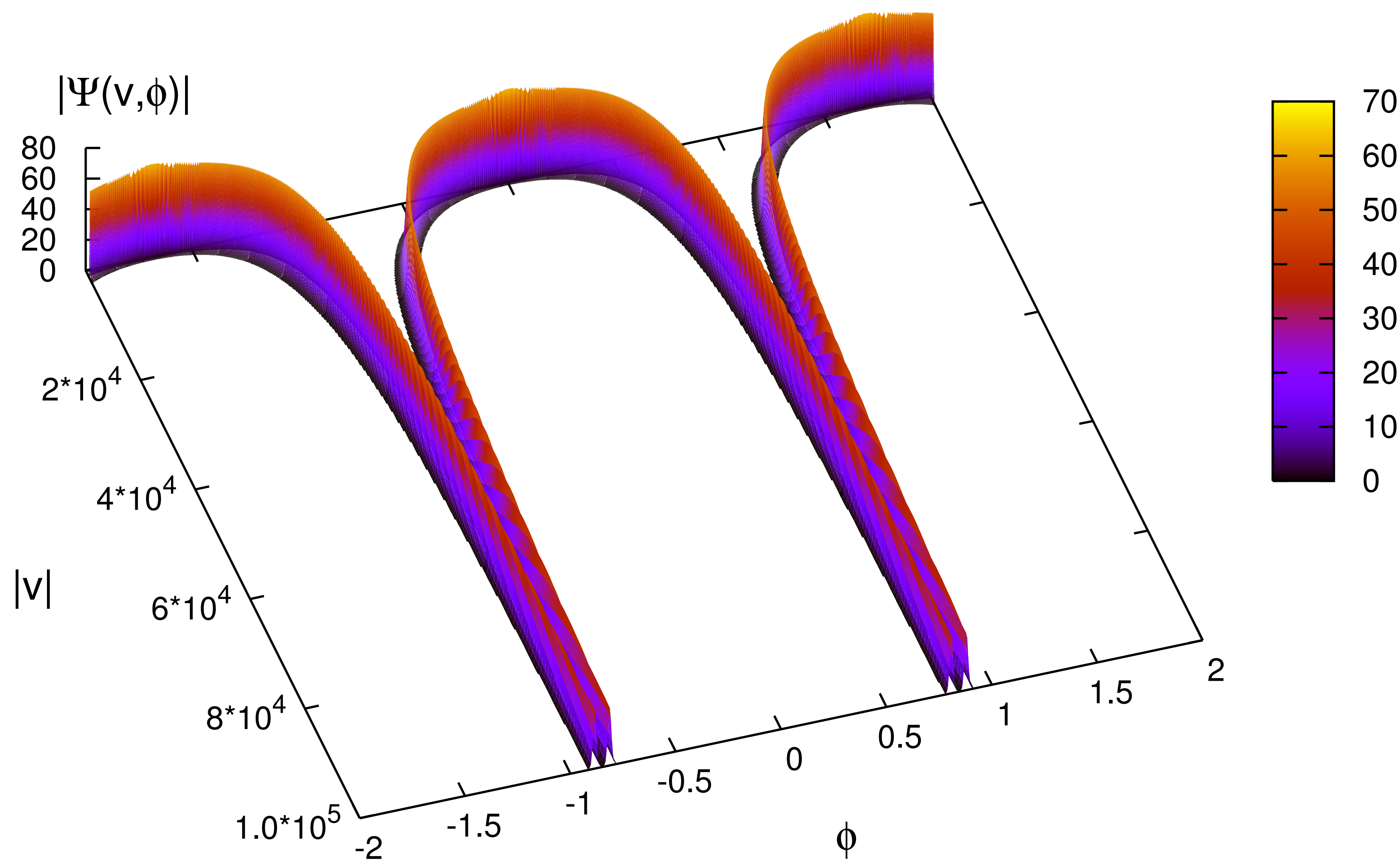


$\Lambda > 0$: *self-adjoint extensions*

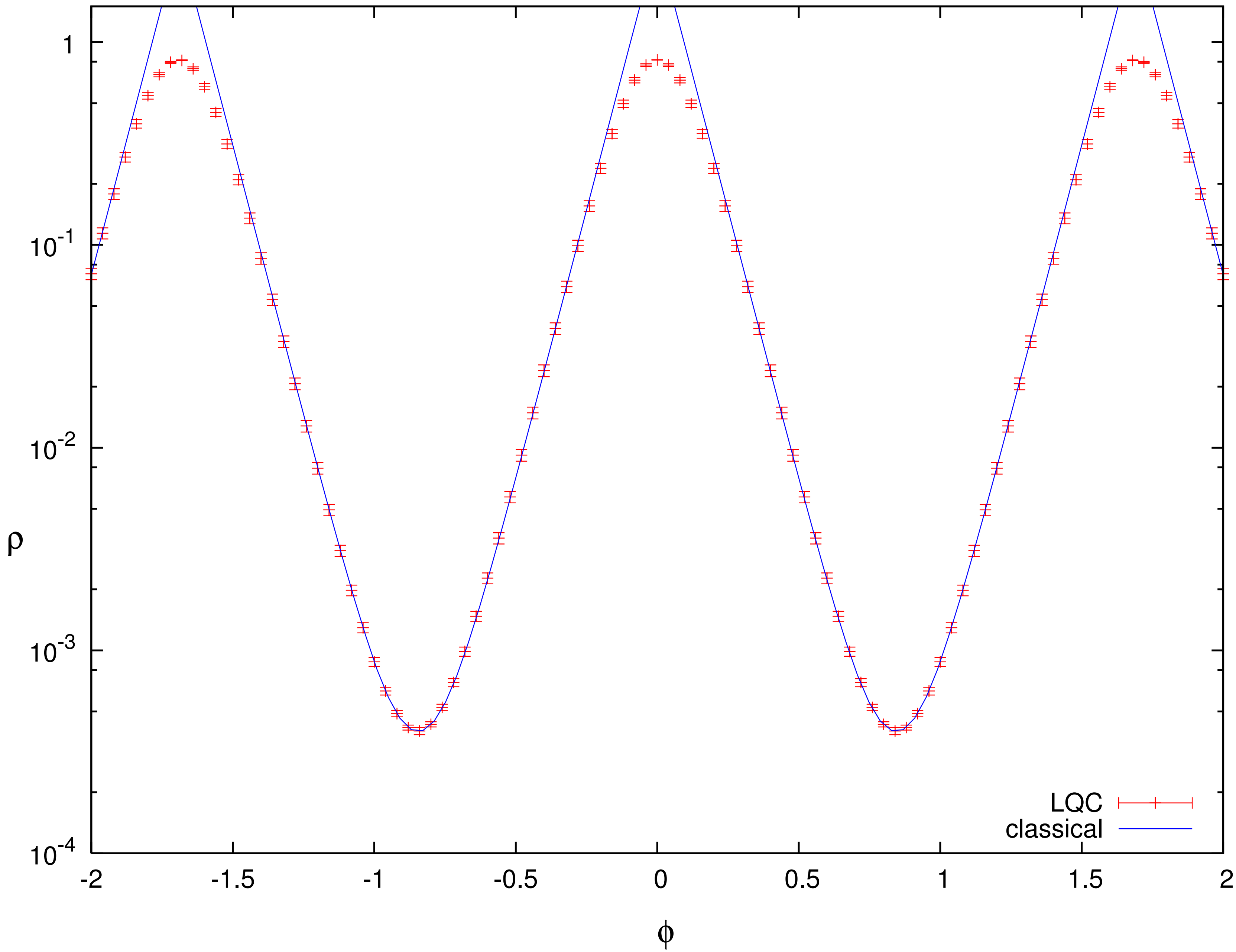
For simplicity we focus on the case $\epsilon = 0$.

- Deficiency subspaces $\mathcal{K}_\pm \subset \mathcal{H}_g$: spaces of normalizable solutions to
$$\langle \varphi_\pm | \, \Theta^\star \mp iI \, | \psi \rangle \; = \; 0, \qquad \psi \in \mathcal{D}$$
found numerically (as solutions to difference equation).
- In symmetric sector solutions unique up to global normalization. $\dim(\mathcal{K}_+) = \dim(\mathcal{K}_-) = 1$ – domain of Θ has many extensions. All of them are defined by unitary transformations $U_\alpha : \mathcal{K}_+ \to \mathcal{K}_-$:
$$\mathcal{D}_\alpha = \{\psi + a(\varphi_+ + U_\alpha(\varphi_+)); \psi \in \mathcal{D}, a \in \mathbb{C}\}$$
- All U_α are of the form $U_\alpha \varphi_+ = e^{i\alpha}\varphi_-$ – 1D family of extensions.
- Since $\mathcal{D}$ – finite combinations of $|v\rangle$ all $\Psi \in \mathcal{H}_{\rm phys}$ have the $v \to \infty$ limit of the form $a(\varphi_+ + U_\alpha \varphi_+)$.
Basis of $\mathcal{D}_\alpha$ – eigenfunctions with this limit.
- Result: All extensions Θ_α of Θ have discrete spectra.
- Physical states have form analogous to ones for $\Lambda < 0$. We can repeat the construction $+$ analysis done for that case.

$\Lambda > 0$: *wave function*



$\Lambda > 0$: *energy density*



$\Lambda > 0$ – *results*

The results are the same for all extensions:

- States remain sharply peaked through the evolution.
- States follow classical trajectory until total energy density approaches critical one, when gravity becomes repulsive and state bounces.
- Bounce joins deterministically contracting and expanding sectors.
- Evolution is nonsingular, bounce replaces singularitites.
- For all extensions the expanding universe after reaching infinite volume (or, equivalently $\rho = \Lambda/8\pi G$) reflects back into contracting one.
- Due to quantum bounce and reflection at infinity we again have cyclic evolution.

Comment:

- Results are analogous for other values of ϵ. For $\epsilon = 2$ one parameter family of extensions. For $\epsilon \neq 0, 2$ – four parameter family.

$\Lambda > 0$ *role of extensions*

- The choice of extension equivalent to selection of reflective conditions at $|v| = \infty$.
- Distinct extensions – different phase rotation at reflection.
- For disemiclassical states trajectories and dispersions same within numerical precision.
  Reason: Existence of unique analytic extension of classical trajectory.
- Bound on Λ: Physical Hilbert space degenerates for $\Lambda > 8\pi G\rho_c$.
  Explanation: Residual energy density above upper bound.
- Dispersion growth:
  - $\forall$ ext. separation of ω_n approaches uniformity similarly to $\Lambda < 0$ case
    $$\omega_n - \omega_{n-1} = \Delta\omega(\Lambda) + O(\omega^{-2})$$
  - Consequence: spreadout of semiclassicall state as slow as for $\Lambda < 0$ (c.a. 10^{70} cycles for dispersions to double).

Alternative picture of $\Lambda > 0$

Work by: W. Kaminski, J. Lewandowski, TP

- APS approach: ϕ used as emergent time, Klein-Gordon like equation, evolution operator has many s.a. extensions. But ...
- Total Hamiltonian constraint C is essentially self-adjoint. Can find $\mathcal{H}^{\rm phy}$ via group averaging. How it is related to extensions ?
- Result of GA: $\mathcal{H}^{\rm phy}$ is a single Hilbert space containing all the extensions

  $$\mathcal{H}^{\rm phy} = \int \mathrm{d}\alpha \tilde{\mathcal{H}}^{\rm phy}_{\alpha}$$

  where $\tilde{\mathcal{H}}^{\rm phy}_{\alpha}$ unitarily related to $\mathcal{H}^{\rm phy}_{\alpha}$
- One can use GA to construct observables (analogs of $\hat{\rho}_{\phi}$). Dynamics not yet analysed ...