Elusive Semi-Classical de Sitter Vacua

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Based on:

Bena, Junghans, Kuperstein, Van Riet, Wrase, MZ (2012)
Gautason, Junghans, MZ (2012)
Blåbäck, Danielsson, Junghans, Van Riet, Wrase, MZ (2010, 2011)

As well as

Wrase, MZ (2010)
Caviezel, Wrase, MZ (2009)
Caviezel, Koerber, Körs, Lüst, Wrase, MZ (2008)
Caviezel, Koerber, Körs, Lüst, Tsimpis, MZ (2008)
Outline

1. de Sitter vacua in string theory
2. Classical de Sitter vacua
3. The Douglas Kallosh problem
4. The validity of the smearing approximation
5. (Anti-)de Sitter from $\alpha'$-corrections?
6. Summary
I. de Sitter vacua in string theory
Perturbative superstring theory in a weakly curved background requires 10 spacetime dimensions.

\[ \mathcal{M}^{(10)} = \mathcal{M}^{(4)} \times_w \mathcal{M}^{(6)} \]

\[ R_c = \text{compact & with small size} \]
Perturbative superstring theory in a weakly curved background requires 10 spacetime dimensions.

\[ M^{(10)} = M^{(4)} \times_w M^{(6)} \]

\( \Rightarrow \) “Compactification”

\( \Rightarrow \) Effective 4D field theory for \( E < \frac{1}{R_c} \)
Moduli fields:

Light 4D scalar fields from higher dimensional field components:
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E.g. metric fluctuations:

\[ \delta g_{MN} \rightarrow \delta g_{\mu \nu}, \delta g_{\mu m}, \delta g_{mn} \]

\[ \mu, \nu = 0, 1, 2, 3 \]

\[ m, n = 4, ..., 9 \]
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• Moduli vevs parameterize background deformations that cost no/little energy
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• \textbf{Two important examples:}

\begin{itemize}
  \item \textit{Volume modulus} $v(x)$ \quad \langle v \rangle \leftrightarrow \text{Vol}(\mathcal{M}^{(6)})
  \item \textit{Dilaton} $\phi(x)$ \quad \langle e^{\phi} \rangle \leftrightarrow g_s
\end{itemize}

\textbf{string coupling}
• Moduli vevs parameterize background deformations that cost no/little energy

• Two important examples:

- Volume modulus \( v(x) \) \( \langle v \rangle \leftrightarrow \text{Vol}(\mathcal{M}^{(6)}) \)
- Dilaton \( \phi(x) \) \( \langle e^{\phi} \rangle \leftrightarrow g_s \)

• Light moduli cause phenomenological problems

(5th force, varying fund. constants, BBN, overclosure,...)

Avoided for \( M_{\text{mod}} \gtrsim (30\text{TeV})^2 \)
Goal: (Meta-)stable de Sitter vacua

V(\varphi)

\varphi^2

\varphi^1
Goal: (Meta-)stable de Sitter vacua

\[ V(\varphi) \]

\[ \varphi^{2} \]

\[ \varphi^{1} \]

\[ \Rightarrow \text{Need } V(\varphi) \text{ with „nice“ local minima } \varphi_{*}: \]

\[ V(\varphi_{*}) > 0 \iff \Lambda > 0 \]

Hessian\((V(\varphi_{*}))\) with eigenvalues \(M_{i}^{2} \geq (30 \text{ TeV})^{2}\)
This is surprisingly difficult!
The effective 4D scalar potential $V(\varphi)$

Starting point: 10D eff. action of massless string states:

$$S_{10D} = S_{\text{Sugra}} + S_{\text{branes}} + S_{\text{pert}} + S_{\text{non-pert}}$$
The effective 4D scalar potential \( V(\varphi) \)

**Starting point:** 10D eff. action of massless string states:

\[
S_{10D} = S_{\text{Sugra}} + S_{\text{branes}} + S_{\text{pert}} + S_{\text{non-pert}}
\]

\[
S_{\text{Sugra}} = \frac{1}{2} \int d^{10}x \left[ \sqrt{-g} \ R(g) + \ldots \right] \leq 2 \text{ derivatives}
\]
The effective 4D scalar potential $V(\varphi)$

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String theory contains **defect-like objects**, e.g.

D(ichlet)-brane \hspace{1cm} O(rientifold)-plane

$T > 0$ \hspace{2cm} $T < 0$
The effective 4D scalar potential $V(\varphi)$

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$$S_{10D} = S_{\text{Sugra}} + S_{\text{branes}} + S_{\text{pert}} + S_{\text{non-pert}}$$

$$S_{\text{brane}} = T \int_{\Sigma_{q+1}} d^{q+1} \xi \ e^{\phi(q-3)/4} \sqrt{-g_{\text{ind}}} + \ldots$$

D(ichlet)-brane

$T > 0$

O(rientifold)-plane

$T < 0$
The effective 4D scalar potential $V(\varphi)$

**Starting point:** 10D eff. action of massless string states:

$$S_{10D} = S_{\text{Sugra}} + S_{\text{branes}} + S_{\text{pert}} + S_{\text{non-pert}}$$

- $S_{\text{pert}} = \text{Perturbative higher derivative corrections}$
  (E.g. Riemann$^n$ -terms)
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From: • **Finite size** effects of the string

(„α‘-corrections“) (α‘ = $l_s^2$)
The effective 4D scalar potential $V(\varphi)$

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(E.g. Riemann $^n$-terms)

From: • Finite size effects of the string
(„$\alpha'$-corrections“) ($\alpha' = l_s^2$)
• String loop corrections

$\sim g_s^{2(L-1)}$
The effective 4D scalar potential $V(\varphi)$

**Starting point:** 10D eff. action of massless string states:

$$S_{10D} = S_{\text{Sugra}} + S_{\text{branes}} + S_{\text{pert}} + S_{\text{non-pert}}$$

$S_{\text{non-pert}}$ : **Non-perturbative** quantum effects:
Instantons, gaugino condensation, ...
The effective 4D scalar potential $V(\varphi)$

Starting point: 10D eff. action of massless string states:

$$S_{10D} = S_{\text{Sugra}} + S_{\text{branes}} + S_{\text{pert}} + S_{\text{non-pert}}$$

„classical“
The effective 4D scalar potential $V(\varphi)$

**Starting point:** 10D eff. action of massless string states:

$$S_{10D} = S_{\text{Sugra}} + S_{\text{branes}} + S_{\text{pert}} + S_{\text{non-pert}}$$

„semiclassical“ or „perturbative“
The effective 4D scalar potential $V(\varphi)$

Starting point: 10D eff. action of massless string states:

$$S_{10D} = S_{\text{Sugra}} + S_{\text{branes}} + S_{\text{pert}} + S_{\text{non-pert}}$$

$$S_{10D} \xrightarrow{\text{Dimensional reduction}} S_{4D} \supset - \int d^4x \ V(\varphi)$$
The effective 4D scalar potential $V(\varphi)$

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$$S_{10D} = S_{\text{Sugra}} + S_{\text{branes}} + S_{\text{pert}} + S_{\text{non-pert}}$$

$$\text{Dimensional reduction} \quad S_{10D} \supset - \int d^4x \; V(\varphi)$$

$$\Rightarrow \quad V = V_{\text{Sugra}} + V_{\text{brane}} + V_{\text{pert}} + V_{\text{non-pert}}$$
The effective 4D scalar potential $V(\phi)$

Starting point: 10D eff. action of massless string states:

$$S_{10D} = S_{\text{Sugra}} + S_{\text{branes}} + S_{\text{pert}} + S_{\text{non-pert}}$$

$\Rightarrow$ $V = V_{\text{Sugra}} + V_{\text{brane}} + V_{\text{pert}} + V_{\text{non-pert}}$
Goal: Construct a simple, fully explicit, and well-controlled de Sitter compactification
2. Classical de Sitter vacua
Simplest attempt: de Sitter vacua from $V_{\text{Sugra}}$

Problem: **No-go theorem**!

Gibbons (1984);
de Wit, Smit, Hari Dass (1987)
Maldacena, Nuñez (2000)
Steinhardt, Wesley (2008)
Simplest attempt: de Sitter vacua from $V_{\text{Sugra}}$

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1) 10D classical supergravity

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   (E.g. $T_{MN} n^N n^M \geq 0$, $n \cdot n = 0$)

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$\Rightarrow$ No de Sitter solutions possible

(Uses just 4D part of 10D Einstein equation)
Manifestation in 4D field theory:

Too steep slope in $v$ whenever $V > 0$
Next to simplest attempt: $V_{\text{Sugra}} + V_{\text{brane}}$

Cf. Silverstein (2007)
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Here: Branes that violate positivity condition of \( T_{MN} \)
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\[ \rightarrow \quad \text{O(rientifold)-planes} \]

\[ T < 0 \]

One finds: \textbf{New no-go‘s unless} \[ \int \sqrt{g} R^{(6)} < 0 \] (negative (integrated) internal curvature)

Hertzberg, Kachru, Taylor, Tegmark (2007)
Silverstein (2007)
10D proof uses \textbf{Einstein} and \textbf{dilaton} equation

4D manifestation:

\begin{align*}
\int d^6x \sqrt{-g} \ R^{(6)} \geq 0
\end{align*}
10D proof uses Einstein and dilaton equation

But for $\int d^6x \sqrt{-g} \ R^{(6)} < 0$:

$$V_{\text{curv}} \propto - \int d^6x \sqrt{-g} R^{(6)}$$

$\Rightarrow$ Use O-planes & negative internal curvature
Problems:

(i) O-planes have partially localized $T_{MN}$

$T_{MN} = 0$

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$T_{MN} \neq 0$

$\Rightarrow$ Complicated gravitational backreaction
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$T_{MN} = 0$  $\Rightarrow$ Complicated gravitational backreaction

$T_{MN} \neq 0$

(ii) Dimensional reduction on negative curvature spaces not well understood

(Zero modes of Laplacian? etc.)
Strategy:

(i) „Smear“ the O-planes:

Localized brane source  \(ightarrow\)  “Smeared” brane source
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Localized brane source $\rightarrow$ “Smeared” brane source

= Common procedure to take backreaction into account in an averaged sense
Strategy:

(i) "Smear" the O-planes:

Localized brane source → “Smeared” brane source

\[ \rho(x) \]

Donnerstag, 10. Mai 2012
(ii) Consider $R^{(6)} < 0$ spaces $M^{(6)}$ that

- yield \textit{supersymmetric} effective actions in 4D
  
  $\rightarrow$ SU(3)-structure manifolds
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(ii) Consider $R^{(6)} < 0$ spaces $\mathcal{M}^{(6)}$ that

- yield **supersymmetric** effective actions in 4D
  $\rightarrow$ **SU(3)-structure manifolds**

- are **group** or **coset** spaces $G/H$

$\rightarrow$ **Left-invariant modes** yield **consistently truncated** 4D supergravity action that can be explicitly computed

**Result:**

Some de Sitter extrema of $V(\varphi)$ have been found

- Caviezel, Koerber, Körs, Lüst, Wrase, MZ (2008)
- Flauger, Paban, Robbins, Wrase (2008)
- Caviezel, Wrase, MZ (2009)
- Danielsson, Haque, Koerber, Shiu, Van Riet, Wrase (2011)

See also:
- Silverstein (2007)
- Haque, Shiu, Underwood, Van Riet (2008)
- Danielsson, Haque, Shiu, Van Riet (2009)
- Andriot, Goi, Minasian, Petrini (2010)
- Dong, Horn, Silverstein, Torroba (2010)
- Danielsson, Koerber, Van Riet (2010)
Problems:

(i) So far all vacua have at least one tachyonic instability

(Saddle points, not minima)
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→ Perhaps just need to scan more examples?
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(ii) *Flux quantization issues*

Danielsson, Haque, Koerber, Shiu, Van Riet, Wrase (2011)
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(iii) Is the smearing really a valid approximation?
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(iii) Is the smearing really a valid approximation?

(iv) The “Douglas-Kallosh problem”
3. The Douglas Kallosh problem
The Douglas-Kallosh problem:

Douglas, Kallosh (2010)
The Douglas-Kallosh problem:

In the absence of warping and higher curvature terms:

Spaces of constant negative curvature require an everywhere negative energy density

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\[ \rho < 0 \]
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But smeared O-planes can provide precisely that!
So where is the problem?
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In the absence of warping and higher curvature terms:

Spaces of constant negative curvature require an everywhere negative energy density.

But smeared $O$-planes can provide precisely that!
So where is the problem?

True $O$-planes are not smeared!
The Douglas-Kallosh problem:

In the absence of warping and higher curvature terms:

Spaces of constant negative curvature require an everywhere negative energy density

\[ R < 0 \]
\[ \rho < 0 \]
So how can negative curvature be sustained if O-planes are localized (as they should be)?
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Note: Is a general issue of negative internal curvature, not necessarily related to dS
Possible ways out: Douglas, Kallosh (2010)

- Everywhere strongly varying warping

(- Or higher curvature terms relevant)
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Varying warping is automatically induced by localized O-planes and D-branes
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  (- Or higher curvature terms relevant)

Varying warping is automatically induced by localized O-planes and D-branes

But if it varies strongly everywhere, it is unclear whether this is still well-approximated by the smeared solution with constant warp factor.
Localized O-plane with everywhere strongly varying warp factor

Smeared O-plane with constant warp factor
4. The validity of the smearing approximation
Our question:

How reliable is the smearing procedure in general?
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1) Do smeared solutions always have a localized counterpart?

2) If yes, how much do their physical properties differ? (e.g. w.r.t. moduli values, cosmological constant,...)
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For 2), cf. also “warped effective field theory”

E.g

- DeWolfe, Giddings (2002)
- Giddings, Maharana (2005)
- Frey, Maharana (2006)
- Koerber, Martucci (2007)
- Douglas, Torroba (2008)
- Shiu, Torroba, Underwood, Douglas (2008)

+ later papers
I) For **supersymmetric** (or **BPS-like**) compactifications

Minkowski vacua à la Giddings, Kachru, Polchinski (2001) + T-dual relatives
Results

Blåbäck, Danielsson, Junghans, Van Riet, Wrase, MZ (2010,2011)

1) For supersymmetric (or BPS-like) compactifications
   Minkowski vacua à la Giddings, Kachru, Polchinski (2001) + T-dual relatives

   • Smearing remarkably robust
     (∆ and moduli vevs unaltered)
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1) For supersymmetric (or BPS-like) compactifications
Minkowski vacua à la Giddings, Kachru, Polchinski (2001) + T-dual relatives

- Smearing remarkably robust
  ($\Lambda$ and moduli vevs unaltered)

- Douglas-Kallosh problem taken care of by warping
  ($\int d^6x \sqrt{-g} \ R^{(6)} < 0$ remains true)
2) For non-supersymmetric compactifications

E.g. $\text{AdS}_7 \times S^3$ with smeared anti-D6-branes and ISD fluxes

Cf. also Saltman, Silverstein (2004)
2) For **non-supersymmetric** compactifications

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- Localized solutions would need **different fluxes**
  
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E.g. $\text{AdS}_7 \times S^3$ with smeared anti-D6-branes and ISD fluxes

Cf. also Saltman, Silverstein (2004)

- Localized solutions would need different fluxes
  ($\Lambda$ and moduli vevs likely to change)
- No continuous interpolation between smeared and localized solution
\(\rho(x)\)
$\rho(x)$
$\rho(x)$
Works for BPS
But:

Only smooth non-BPS solution is the smeared one:

\[ \rho(x) \]
Moreover:

If a localized solution disconnected from the smeared one exists, it must involve non-standard boundary conditions at the D6-brane (divergent $H_3$).

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If a localized solution disconnected from the smeared one exists, it must involve non-standard boundary conditions at the D6-brane (divergent $H_3$).


Whether this makes sense is still unclear

Cf. also Blåbäck, Danielsson, Van Riet (2012)

Bena, Grana, Halmagyi (2009)
Inconclusive, but control issues become already quite non-trivial at this level
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Even if successful, purely classical de Sitter compactifications most likely have large $\Lambda$

(No naturally small parameters involved)
5. (anti-)de Sitter vacua from $\alpha'$-corrections?
Next-to-Next-to simplest approach (?):

\[ V_{\text{Sugra}} + V_{\text{branes}} + V_{\text{pert}} \]
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\[ V_{\text{Sugra}} + V_{\text{branes}} + V_{\text{pert}} \]

A case study: Heterotic string at string tree-level

- \( \alpha' \)-corrections already at order \((\alpha')^1\)
- Are completely known at that order
- Includes Riemann\(^2\)-term
Next-to-simplest approach (?):

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- \( (\alpha')^n \)-corrections not known explicitly, but „sufficiently“ constrained  
  (see below)
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A case study: *Heterotic string at string tree-level*

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  (see below)

**Hope:** Small \(\Lambda\) as subleading effect?
Recent work: Green, Martinec, Quigley, Sethi (2011)

At order $\alpha'$:

- **No de Sitter vacuum possible**
- **But: Possible AdS-vacua with small $\Lambda \sim \alpha' C$**
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At order $\alpha'$:

- **No de Sitter** vacuum possible
- **But**: Possible **AdS**-vacua with small $\Lambda \sim \alpha' \, C$ ?

$$\mathcal{C} = \frac{1}{2V'} \int d^6y \sqrt{\tilde{g}_6} e^{6A - \frac{\phi}{2}} \left\{ 3(\partial \omega)^4 + 2[(\partial_m \omega)(\partial_n \omega) - \tilde{\nabla}_m \partial_n \omega - \tilde{g}_{mn}(\partial \omega)^2]^2 + \frac{1}{2} e^{-4\omega} [H_{mn} \partial_l \omega]^2 \right\}$$

$$\mathcal{V}' = \int d^6y \sqrt{\tilde{g}_6} e^{6A}$$

$$\omega = A + \frac{\phi}{4}$$
Recent work: Green, Martinec, Quigley, Sethi (2011)

At order $\alpha'$:

- **No de Sitter vacuum possible**
- **But: Possible AdS-vacua with small $\Lambda \sim \alpha'$ (...)**

$\Rightarrow$ Preference of **AdS over dS even under inclusion of higher derivative terms?**
Careful study: Gautason, Junghans, M.Z. (2012)

No perturbatively small $\Lambda \sim \alpha'^n$ (...) is possible, no matter what the sign is!

Cf. also Held, Lüst, Marchesano, Martucci (2010)
Sketch of argument:

\[ S_{4D} = \int d^4x \sqrt{-g_4} \{ R_4 - V + W \} \]

\[ V = e^{-2\phi}(\ldots) \]
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\underline{Dilaton eq.} \hspace{1cm} 2V - \partial_\phi W = 0
Sketch of argument:

\[ S_{4D} = \int d^4x \sqrt{-g_4} \left\{ R_4 - V + W \right\} \]

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Dilaton eq.
\[ 2V - \partial_\phi W = 0 \]

Einstein eq.
\[ R_4 - 2V - W' = 0 \]

\[ W' \equiv \frac{g^{\mu\nu}}{\sqrt{-g_4}} \frac{\delta}{\delta g^{\mu\nu}} \left( \int d^4x \sqrt{-g_4} \ W \right) \]
Sketch of argument:

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\]

\[
V = e^{-2\phi}(\ldots)
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\]

\[
R_4 = \partial_\phi W + W' \sim \sum_{m,n} \alpha' m \text{ Riemann}^n
\]
Sketch of argument:

\[ S_{4D} = \int d^4x \sqrt{-g_4} \{ R_4 - V + W \} \]

\[ V = e^{-2\phi}(\ldots) \]

Dilaton eq. \hspace{1cm} 2V - \partial_\phi W = 0

Einstein eq. \hspace{1cm} R_4 - 2V - W' = 0

\[ R_4 = \underbrace{\partial_\phi W + W'}_{\sim \sum_{m,n} \alpha'^m \text{ Riemann}^n} \]

\[ \Lambda = \sum_{m,n>0} c_{mn} \alpha'^m \Lambda^n \]
Sketch of argument:

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S_{4D} = \int d^4x \sqrt{-g_4} \{ R_4 - V + W \}
\]

\[
V = e^{-2\phi} (\ldots)
\]

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\[
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\]

If \[
\Lambda = \Lambda_0 + \alpha' \Lambda_1 + \alpha'^2 \Lambda_2 + \ldots
\] \Rightarrow \[
\Lambda = 0
\]
However:

AdS-solutions of $O(\alpha')$-action with large $\Lambda \sim 1/\alpha'$

Lechtenfeld, Nölle, Popov (2010)
Chatzistavrakidis, Lechtenfeld, Popov (2012)
6. Summary
• Classical or semi-classical de Sitter vacua are surprisingly difficult to build

• Explicit well-controlled examples hard to come by

• Most popular scenarios are less explicit and involve non-perturbative quantum corrections
  E.g. KKLT; LARGE Volume; M-theory on G; heterotic orbifolds/Calabi-Yau;...

Maybe de Sitter vacua are only possible in computationally challenging regimes... 😞

Cf. Dine, Seiberg, 1985
A general issue:

**Naively:** For $N$ scalars, the probability of no tachyons in a de Sitter extremum is $P \sim 2^{-N}$.
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More sophisticated estimates:

- Marsh, McAllister, Wrase (2011)
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Typically: $N \sim O(10) \ldots O(100)$

**Few** de Sitter extremum are **local minima**?