Progress on dS/dS + FRW Holography

Based on works with Alishahiha, Dong, Horn, Karch, Matsuura Polchinski, Tong, Tomoba (2004 - 2012)

0. brief intro/review of framework

1. Unitarity bounds
   t-dependent (dual) QFT
   Dong Horn ES Tomoba '12

2. UV structure of dS/dS + holographic RG

3. additional comments & questions
Start by asking what happens to AdS/CFT when we uplift to cosmology in concrete examples.

Goal: Make holographic cosmology a precision science... with understanding of error bars along the way.
AdS/cFT, brane construction

\[ \left(\frac{dR}{dr}\right)^2 = \frac{1}{R^2} \rightarrow R = r \]

\( R > 0 \) Einstein space

Color branes

Gravitational Redshift \( g_{00} \approx 1 - \frac{R^4}{r^4} \)

= Low-energy region

Our Strategy:

\[ E = \sqrt{-g_{00}} \quad E_{pr} \ll E_{pr} \]

\rightarrow Effective field theory (EFT)

dual, complete QFT

in strict near-horizon limit

Note: CFT does not live on the boundary.
RS/warped compactifications

\[ ds^2 = \frac{r^2}{R^2} \, dx^2 + \frac{R^2}{r^2} \, dr^2 + \text{internal} \]

\[ r < r_{uv} \]

H. Verlinde, Giddings/Kachru/Polchinski

warped throat

\[ E < \Lambda_c = \frac{r_{uv}}{R^2} \sqrt{M_p^2 - \frac{r_{uv}^2}{R^4} N^2 + \frac{\text{Vol}(X)}{g_s^2}} \]

\[ \rightarrow \text{complete QFT in limit} \]

\[ r_{uv} \rightarrow \infty \text{ with } \frac{r_{uv}^2 N^2}{R^4} > \frac{\text{Vol}(X)}{g_s^2} \]

(Can happen in t-dependent way.)

Turns out to be a good analogy for

\[ ds \rightarrow \text{FRW} \]
Uplifting AdS/cFT: Brane Constructions

\[ ds^2 = \sin^2 \frac{w}{L} ds^2_{D-1} + dw^2 \]

2 redshifted regions

\[ (\frac{dR}{dr})^2 = \frac{-1}{R^2} + \frac{\text{const}}{R^{n+2}} \]

\( dS_D \equiv 2 \ \text{EFTs}_{D-1} \ \text{w/GR}_{D-1} \)

We'll specify UV behavior shortly.

FRW also has 2 redshifted regions, and GR_{D-1} decouples at late times.

\( S \to \infty \)

\( t \)-dependent (power law) couplings

\( \Rightarrow \) matter content via unitarity

\( \bullet \) from flavors
We can be more precise:

\[
\frac{ds^2}{ds_D} = \sin^2 \frac{w}{l} ds^2_{d-1} + dw^2
\]

\[
Z_{\text{bulk}} = \int D\tilde{\Phi} \int [D\Phi] \left< e^{-iS} \Phi(v) e^{iS} \Phi(v') \right>
\]

\[
\left| Z_{QFT}^{(\text{EL})} \right>< \left| Z_{QFT}^{(\text{EL})} \right>
\]

\[
Z_{QFT}^{(\text{EL})} \quad \text{and} \quad Z_{QFT}^{(\text{EL})}
\]

e.g. at Gaussian level,

\[
\frac{\langle O O \rangle}{g^2} = \delta^2 \overline{Z_1} = \frac{\Gamma \left[ \frac{1}{2} (n-d-\hat{\Delta}+\ell) \right] \Gamma \left[ \frac{1}{2} (1+\hat{\Delta}+\ell) \right]}{\Gamma \left[ \frac{1}{2} (n-\hat{\Delta}+\ell) \right] \Gamma \left[ \frac{1}{2} (\hat{\Delta}+\ell) \right]}
\]

\[
\hat{\Delta} = \hat{d} + \sqrt{(\hat{d})^2 - 4m^2} \quad \text{(but} \langle O O \rangle \text{ real)}
\]

\[
\text{cf Boussou Malmoey Strominger}
\]

\[
\hat{d} \text{ flows to} \quad \frac{d}{2} + \frac{1}{2} \text{ in UV}
\]

\[
\text{encodes (max. symmetric) shape of } dS_D \text{ warp factor}
\]

\[
\text{Holographic RG in progress (result below...)}
\]
Comparative holography:

Recall in AdS/CFT that p-brane construction lands on Poincare' slicing. In dS + FRW, above brane constructions $\rightarrow$ slicings

\[ ds/\text{d} s \quad \text{and} \quad \text{FRW/t-dependent QFT} \]

- inside a causal region
- a spatial direction ($\leftrightarrow$ scale) emerges.

$\Rightarrow$ - # of degrees of freedom real, $>0$
and unitarity more transparent
- symmetries (such as they are) less manifest

As in AdS $\rightarrow$ Global, this may connect
to other slicings (dS/CFT, FRW/CFT)

* Must make sense of $\mathcal{S}D_{\mu} \mathcal{D}_{\nu}$

Harlow/shenker/Stanford/Susskind '12

\[ \text{Anninos/Hartman/Stramner} \]
We would like to extract the essential features of the dual theories, given the concrete brane constructions.

Plan:

1. Flavor content of FRW duals, unitarity, & time-dependent QFT couplings
2. Comments on Structure of $\text{AdS}_D$ duals
Magnetic flavors + uplifting

e.g. IIB (p,q) 7Bs wrapping $\Sigma_3 \subset S^5$

Tension $\sim \frac{N}{R^2} \frac{1}{g_s}$

$S^1 \rightarrow S^5 \alpha^3 \downarrow ClP^2 \alpha^1$

Competes with curvature

on $CP^1$: $24$ 7Bs $\rightarrow R = 0$

* $CP^2$: $36$ 7Bs $\rightarrow R = 0$

Banks, Douglas, Seiberg, Aharony, Maldacena, Fayaztudin
Joe PES

$\Delta N \equiv N - N_{R=0}$

$\Delta N < 0$ | $\Delta N > 0$

Ads | Cosmo

Color branes \hspace{1cm} flavr branes

magnetic flavors \hspace{1cm} static sol'n

$\Delta N \geq 0$ distinction in dual QFT?
Unitarity bounds & \( t \)-dependent QFT.

Given well-defined QFT at some scale, unitarity bounds help constrain IR physics.

\( \Delta \text{ scalar} > \frac{d-2}{2} \) (e.g. \( \Delta \text{ scalar} > \frac{d-2}{2} \))

\( N = 1 \) SQCD \( N_c \) colors \( + N_f \) flavors \( Q, \bar{Q} \)

\( \text{IF SCFT: } \Delta_{\text{chiral}} = \frac{3}{2} |R| \)

\( \text{unitarity } \Rightarrow \Delta \geq \frac{d-2}{d-2} = 1 \)

\( \Delta(Q\bar{Q}) = \frac{3(N_f - N_c)}{N_f} \geq 1 \Rightarrow N_f \geq \frac{3}{2} N_c \)
In D3 - (p,q)7 system,
\[ \Delta n > 0 \implies \text{no static solution.} \]

* In the simplest case (with parallel 7-branes \( \Rightarrow N=2 \) susy)
this follows from unitarity.

Seiberg-Witten curve
\[ y^2 = x^3 - f(u)x - g(u) \]
(since)
\[ 2\Delta y = 3\Delta x = 5\Delta u \]

\[ \frac{d\lambda_{SW}}{du} = \frac{dx}{dy} \quad \alpha_u \int_{\lambda_{SW}} \frac{udx}{y} \]

BPS masses \( \Rightarrow \begin{cases} \Delta u = (1 + \Delta y) \\ \Delta x \end{cases} \)
\( \text{dim 1} \)

\[ \Rightarrow \Delta u = \frac{12}{12 - N_f} \quad \text{, } N_f > 12 \]

would violate unitarity.
But there do exist $t$-dependent solutions with the required properties (redshift, $N_{\text{ dof }}$, $M_p \to \infty$) for a dual EFT $\to$ QFT.

More general question: how do $t$-dependent couplings affect unitarity bounds on IR behavior $\leftrightarrow$ field content
Consider
\[ \mathcal{S} \mathcal{O} \mathcal{L} = \int dt \, d^{d-1} x \, g(t, x) \mathcal{O} \]
\[ g = g_0 \, t^y \quad \alpha \quad g_0 (t^2 - x^2)^{\frac{\nu}{2}} \]
as \[ t \to \infty \]

- \( \alpha \) can change whether
- \( \Delta L \) dominates at late times (IR)
- IF \( \Delta L \) marginal in IR
under \( x^\nu \to 1x^\nu \), then
\[ [\mathcal{O}] = d + \alpha \]

\[ \rightarrow \text{Expect } \alpha \text{ can shift relevance condition & unitarity bounds} \]
We can analyze this explicitly in large-$N$ double trace flows.

\[
\Delta S = \int \frac{1}{2} \phi m^2 \phi + g(t, \vec{x}) \phi^2
\]

\[
= \int \frac{g^2}{2m^2} \phi^2
\]

\[
\langle \phi \phi \rangle = \ldots + \frac{1}{\sqrt{N}} + \text{non-adiabatic effects}
\]

Effectively Gaussian
Static Limit:
\[
\Delta_{\pm} = \frac{1}{2} \pm \nu
\]
\[
\langle \mathcal{O}_+ (p) \mathcal{O}_- (-p) \rangle = -i \, C_{\pm \nu} (p^2 - i \varepsilon)^{\pm \nu}
\]

\[
S_{\text{CFT}}^{(+)} + \int \frac{g^2}{4m^2} \phi \phi^\dagger
\]

\[
\Delta = d + 2\nu
\]
irrelevant

\[
\langle \phi(p) \phi(-q) \rangle = -i \int \frac{d^2(p-q)}{m^2 - g^2 C_\nu (p^2)^{\nu}}
\]
\[
\rightarrow \langle 0 | 0 \rangle
\]
as \( p^2 \to \infty \)
\([t\text{-dependent case}]\)

- \[\int \lambda_0 t^{2\phi} \Theta_+^2 \text{ is relevant}\]
  
  (dominates 2 pt fnns at large \(\Delta t\))

  when \([\lambda_0] = 2(g^2 - \nu) > 0\)

- Unitarity maintained, including \(\nu > 1\)

- Can UV complete, e.g. SUSY models
  
  (effects of \(\lambda\) lost for \(\Delta t < \frac{\lambda}{\Delta}\))
\[ \Delta S = \int \frac{1}{2} \phi m^2 \phi + g(t, x) \phi \phi \]

\[ \left\langle \phi(p) \phi(q) \right\rangle \rightarrow i \delta(p-q) \frac{(p^2 - i \varepsilon)^{-\nu}}{C_\nu} \]

\[ \Rightarrow \left\langle \phi(x) \phi(x') \right\rangle = \frac{-1}{C_\nu < v g(x) g(x') (x-x')^{2 \Delta}} \]

*Despite the $\Delta$, here, forward scattering is unitary*

\[ \text{Im} A(x \rightarrow x) < -\sin \pi nu/C_\nu > 0 \]

*(Trinizalizer/Ginzburg: \text{Im} A_{\phi} < \phi (\Delta - (d-2)) \text{ in CFTs})*
Note: can be more relevant than mass term, maintain long-range correlations.

Scales in $t$-dependent case:

$\int \frac{d^2 \theta}{\pi} \theta^2 \text{ relevant & unitary}$

=> UV complete...

SUSY gauge theory $\theta$ composite below $\Lambda$

- $\alpha(t)$ dynamical:

... or cut off

$\frac{1}{\sqrt{t}}$ would get $\frac{1}{\sqrt{t}}$ from $V$

L would not matter.

$1$-dependence of $t$. I doesn't matter.
In $dS_3^*$, we expect a similar effect:

- Heavy flavor branes alone non-unitary (static)
- Correct the metric on Coulomb branch $ightarrow$ restore unitarity

(in progress on QFT side of explicit uplifted $dS_3$ construction)
Techniques:

- AdS $\times S \times T^n$

"uplift" curvature energy via variation of $T^n$ (axio-dilaton, cf F theory) over the original Freund-Rubin base.

\[ \text{vary size and/or shape} \quad (p \text{ or } T) \]

(can use e.g. SUSY sigma model to describe fibration, with motion of "stringy cosmic branes" described appearing in the superpotential)

$T^n$ fibration can consistently cancel (CY), under-cancel, or over-cancel the curvature of the base.)
• Orientifolds provide crucial negative term in moduli potential.

$$d s^2_{0\text{-plane}} = dx^2_\perp (1 - \frac{r_0^n}{r^n}) + dx^2_\parallel \frac{1 - r_0^n}{r^n}$$

counteracts deficit angles introduced by elliptic fibration.

In a back reaction:

• Insist on perturbative control
  • radii $\gg \sqrt{\phi}$
  • bound/incorporate warp factor gradients cf Giddings, Douglas, D. + Kallosh, Maharana, Torroba de Wolfe...
A concrete example

\[ dS_3 = dS_2 + \text{large-}c \text{ matter} \]

<table>
<thead>
<tr>
<th>Colors ( D1 )</th>
<th>( D5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma )</td>
<td>( \Sigma )</td>
</tr>
<tr>
<td>( \times \times )</td>
<td>( \times \times )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fibration ( \Sigma P5 )</th>
<th>( \Sigma P5' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \times \times )</td>
<td>( \times \times )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Negative term ( \Sigma 05 )</th>
<th>( \Sigma 05' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \times \times )</td>
<td>( \times \times )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NS ( \Sigma )</th>
<th>( \Sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \times \times )</td>
<td>( \times \times )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NS' ( \Sigma )</th>
<th>( \Sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \times \times )</td>
<td>( \times \times )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Or ( D5s ) { ( D7-\bar{7} ) }</th>
<th>( D7-\bar{7}' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \times \times \times \times )</td>
<td>( \times \times \times \times )</td>
</tr>
</tbody>
</table>

(Most pairwise SUSY)
Potential for $\beta = \frac{kR_\ell}{R}$, $R$, $L$, $g_s$:

$$\tilde{\eta} = \frac{g_s}{R^2 L^2}$$

$$\rho = b_T + i L^2$$

$$\mu \approx 16 M_3^3 k^3 \left\{ \left( 4 \pi^2 - \frac{2 \pi^2}{3 \beta^2} \right) \left[ 24 - n_{\rho} - \tilde{n}_{\rho} \left( \log \left( \frac{L^2}{L^*} \right)^2 + \left( \frac{b_T - b_5}{L^4} \right)^2 \right) + \frac{\pi k n_{NS5}}{L^2 \beta^3} \right] \right\} \tilde{\eta}^4$$

$$- \left( 2 \pi R^2 - \frac{n_D T R^4}{2 k} \right) \frac{\tilde{\eta}^5}{\beta^3} + 4 \pi^2 \left( N_{D5}^2 L^4 + \frac{(N_D + b_7 N_{D5})^2}{L^4} + 2 b_7 N_{D5}^2 \right) \frac{k\tilde{\eta}^6}{\beta^4}$$

- $n_{\rho} = 4 = \# \text{ of } \rho 5\text{-branes}$
- $n_{\rho}^J = 2 = \# \text{ of stacks of } \rho 5\text{-branes}$

(explicit GLSM construction of this fibration)

* curvature $R = \frac{1}{R^2}$: require $g' R << 1$

($10^{-2}, 10^{-3}$)

small $R_\ell, L \not\Rightarrow$ large curvature
• The ingredients are pairwise SUSY, and source small warp factor or have known back reaction (e.g. elliptic fibration).

⇒ $U$ given above is a good approximation given a solution with $g_s << 1$, $g' g R << 1$

• Also address other directions in scalar field space (e.g. axions, anisotropies)

* aside from $Dp - \overline{Dp}$: these are stabilized by Wilson lines
Metastable dS3, with parametric estimate for Gibbons-Hawking entropy

\[ S \sim M_3 R_{dS} \sim \frac{R_f R^2 L^4}{g_5^2} \cdot R_{dS} \]

\[ \sim k N_{D1} N_{D5} \frac{\epsilon^{3/2}}{\epsilon^{3/2}} \]

(parametric count of horizon degrees of freedom)

\( \epsilon^{-3/2} \) factor: cf. Polchinski-ES '09

from hierarchy \( R_{dS} \sim \frac{R^2}{\epsilon} \)

\( \# \) light winding strings

\( \sim (R_{dS}/R)^3 \sim \frac{1}{\epsilon^{3/2}} \)
Further Comments on $dS_3$ duals:

- Non-perturbative instabilities

Our explicit $(A) dS_3$ vacua are all metastable

In $AdS_3 \Rightarrow$ no dual CFT at $d >> 1$, but long-lived with cutoff (e.g. warped compactification). Similar to

$AdS \times S^5 / \mathbb{Z}_k$

( susp with (free action) Quiver $U(N)^k$

Non-perturbative instability to develop VEVs.

Kachru ES, ... Horowitz / Gava / Polchinski, Simić / Tavšič
\[
\frac{ds^2}{D} = \sinh(h)^2 \frac{w}{L} ds_{D-1}^2 + dw^2
\]

In \textit{AdS}_D, \textit{Moduli fixing is dual to } \beta_{\{1\}} = 0.

In \textit{dS}_D, \textit{we also fix the moduli (more strongly, since no allowed tachyons)}.

Recent (in) progress: \underline{holo}graphic RG

... Heemskerk/Polchinski

find \( V' = 0 \) \(\Rightarrow\) coefficient of single-trace terms in Wilsonian action do not depend on scale \( \ell \), for both \( A/\text{i} ds_0/\text{d}s_0 \).
Recall
\[ ds^2 = \sin^2 \frac{\omega}{L} ds_d^2 + dw^2 \]

\[ Z_{\text{bulk}} = \int D\Phi \int [D\Phi] \left| e^{iS} \right| = \int [D\Phi] e^{iS} \]

\[ \left. \Phi(x) \right| = \frac{1}{\hat{\Delta}} \]

\[ Z_{\text{QFT}} \]

\[ Z_{\text{QFT}} \]

\[ Z_{\text{Planck}} \]

\[ \langle \Phi \Phi \rangle = \frac{Z_{\text{Planck}}}{Z_{\text{QFT}}} \]

\[ \frac{Z_{\text{Planck}}}{Z_{\text{QFT}}} = \frac{\Gamma\left[ \pm \left( h_d - \hat{\Delta} + l \right) \right] \Gamma\left[ \pm \left( 1 + \hat{\Delta} + l \right) \right]}{\Gamma\left[ \pm \left( d + \hat{\Delta} + l \right) \right] \Gamma\left[ \frac{\hat{\Delta} + l}{2} \right]} \]

\[ \hat{\Delta} = \frac{d}{2} + \sqrt{\frac{d}{2} - M_0^2} \quad \text{but} \quad \langle \Phi \Phi \rangle_{\text{g.s.}} \text{ real} \]

\[ \hat{\Delta} \text{ flows to } \frac{d}{2} + \frac{1}{2} \text{ in UV} \]

\[ \text{encodes (max. symmetric) shape of } dS_0 \text{ warp factor} \]
A more general construction?

We can reproduce $\mathbb{Z}[\hat{\phi}]$, e.g.

$$
\left< \phi \phi \right>'' = \frac{\delta^2 \mathcal{Z}_1}{\delta \phi^2} = \frac{\Gamma \left[ \frac{1}{2} (H_d - \hat{\Delta} + L) \right] \Gamma \left[ \frac{1}{2} (1 + \hat{\Delta} + L) \right]}{\Gamma \left[ \frac{1}{2} (d - \hat{\Delta} + L) \right] \Gamma \left[ \frac{\hat{\Delta} + L}{2} \right]}
$$

by any

$$
S_{\text{FT}} + \sum_\ell \hat{u}_\ell (\delta \ell) (\delta \ell) + \ldots
$$

no relevant operators e.g. deWolfe Giampietri Kachru Taylor

$$
\hat{K} + \left< \phi \phi \right>_{\text{FT}} = \frac{1}{\left< \phi \phi \right>''} ; \text{ similarly solve non-local...}
$$

By construction, produces $\mathcal{D} \mathcal{S}$, results at semiclassical level... potential problems at $\frac{1}{N}$, a UV completing $\mathcal{S}$
Summary So far

- Can uplift AdS/CFT to cosmology
  - 2 EFT's + GR_{D-1}
    - Concrete brane constructions decouples at late time

- Magnetic flavor content surviving at low energies, \( N_f > N_{cft} \)
  - Consistent with basic unitarity checks.

- Can go beyond low energy description, e.g. capturing the \( dS_D \rightarrow dS_{D-1} \), warp factor
Other directions

- Flux vacua: trade flux for branes to expose dual d.o.f.
  cf. $N=4$ SYM Coulomb branch
  e.g. KKLT: reproduces $SU(N)_b$ and $\frac{C_3}{C_5'} = \frac{N'}{N}$

- Spherical slicing of static patch:
  (Aminos, Hartnoll, Hofman '11)
  build up from $AdS_2 \times S^2 \times CY$, another opportunity for concrete brane construction.