Brane Backreaction: antidote to no-gos

Getting de Sitter (and flat) space unexpectedly

w Leo van Nierop
Outline

- New tool: high codim back-reaction
  - RS models on steroids
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  - RS models on steroids
- Honest-to-God higher-dim cosmology
  - de Sitter no-go and inflationary models
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• New tool: high codim back-reaction
  • RS models on steroids

• Honest-to-God higher-dim cosmology
  • de Sitter no-go and inflationary models

• Implications for naturalness issues
  • Hierarchy problem
  • Cosmological constant problem
New Tools

arXiv:0705.3212 w Hoover & Tasinato
arXiv:0812.3820  w Hoover, de Rham & Tasinato
arXiv:0912.3039 w Bayntun and van Nierop
Extra dims and naturalness

- How can extra dimensions help?
  - scalars in 4D need not be scalars in higher D

\[ \text{eg } \phi \text{ could arise as a component of } A_m \text{ or } g_{mn} \]
Extra dims and naturalness

• How can extra dimensions help?
  • scalars in 4D need not be scalars in higher D

• lowering the gravity scale helps by lowering the UV scales to which one can be sensitive

\[ \delta m^2 = \frac{\Lambda^4}{M_4^2} \]
Extra dims and naturalness

- back-reaction can be important for low energies (eg Randall-Sundrum models)

Cod-1 back-reaction (warping) makes low E physics local in extra dims

Worked out in detail for codimension-1
Why are higher codimensions harder?

- In $d$ space dims massless fields vary as $r^{2-d}$ and so tend to diverge at the source positions for $d > 1$
Higher-codimension back-reaction

• Why are higher codimensions harder?
  • In $d$ space dims massless fields vary as $r^{2-d}$ and so tend to diverge at the source positions for $d > 1$

• How is this dealt with?
  • Source action dictates near-source boundary conditions
Higher-codimension back-reaction

- Why are higher codimensions harder?
  - In $d$ space dims massless fields vary as $r^{-d-2}$ and so tend to diverge at the source positions for $d > 1$.

- How is this dealt with?
  - Source action dictates near-source boundary conditions:
    $$\int E \cdot dA \rightarrow Q$$
Higher-codimension back-reaction

Why are higher codimensions harder?

- In $d$ space dims massless fields vary as $r^{d-2}$ and so tend to diverge at the source positions for $d > 1$.

How is this dealt with?

- Source action dictates near-source boundary conditions.

\[ r^{d-1} \frac{\partial \phi}{\partial r} \to \frac{\partial S}{\partial \phi} \]

...and similarly for $g_{mn}$ etc.

Peloso, Sorbo & Tasinato
CB, Hoover, de Rham & Tasinato

Penn State May 2012
Codimension-2 back-reaction

• *eg: for scalar-tensor theories*

given the action:

\[
S = \frac{1}{2\kappa^2} \int \sqrt{-g} \left[ R + (\partial\phi)^2 + V(\phi) \right] + S_b
\]

and

\[
ds^2 = e^{2W} g_{\mu\nu} dx^\mu dx^\nu + dr^2 + e^{2B} d\theta^2
\]

with

\[
S_b = \int \sqrt{-g} L_b
\]
Codimension-2 back-reaction

Then the matching conditions are

\[
(e^B \phi')_b = \frac{\kappa^2}{2\pi} \left( \frac{\partial L_b}{\partial \phi} \right)
\]

\[
(e^B W')_b = \frac{\kappa^2}{4\pi} \left( \frac{\partial L_b}{\partial g_{\theta\theta}} \right) = U_b
\]

\[
(e^B B' - 1)_b = -\frac{\kappa^2}{2\pi} \left[ \left( \frac{\partial L_b}{\partial \phi} + \frac{3}{2} \frac{\partial L_b}{\partial g_{\theta\theta}} \right) \right]
\]

Constraint: \( 4U_b [2 - 2L_b - 3U_b] - \left( \frac{\partial L_b}{\partial \phi} \right)^2 = 0 \)
Higher-codimension back-reaction

- Important complication
  - Divergences due to fields diverging at branes absorbed into renormalization of brane interactions
Higher-codimension back-reaction

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- A convenient regularization
  - Codimension-1 cylinder with smooth interior
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- Matching rules are robust
  - Can use any brane regularization at all to capture lowest multipole moment
Higher-codimension back-reaction

- Important complication
  - Divergences due to fields diverging at branes absorbed into renormalization of brane interactions
  - A convenient regularization
  - Codimension-1 cylinder with smooth interior
- Matching rules are robust
  - Can use *any* brane regularization at all to capture lowest multipole moment

Check: the result works well for D7s sourcing axio-dilaton and gravity in 10D Type IIB sugra

Baytun, CB & van Nierop
Green, Shapere, Vafa & Yau

Penn State May 2012
de Sitter solutions

hep-th/0512218 w Tolley, Hoover & Aghababaie
arXiv:1109.0532 w Maharana, van Nierop, Nizami & Quevedo
Back-reaction vs de Sitter no-go

- Few honest-to-God extra dimensional inflationary cosmologies exist
  - work in 4D effective theory
  - work with moving branes in static backgrounds
Back-reaction vs de Sitter no-go

- Few honest-to-God extra dimensional inflationary cosmologies exist
  - work in 4D effective theory
  - work with moving branes in static backgrounds
- Lower-dimensional dS solutions in higher dimensions difficult to find
  - Motivated no-go results for de Sitter solutions
Back-reaction vs de Sitter no-go

CB, Maharana, van Nierop, Nizami & Quevedo

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**de Sitter solutions and no-go results:**

\[
S = \frac{1}{2\kappa^2} \int \sqrt{-g} \left( R + L_m \right) + S_{\text{source}}
\]

and

\[
ds^2 = e^{2W} g_{\mu\nu} dx^\mu dx^\nu + \hat{g}_{mn} dx^m dx^n
\]

then in absence of space-filling fluxes

\[
\frac{1}{2\kappa^2} \int R = S_{\text{on-shell}} + \int \nabla^2 e^{dW} + g_{\mu\nu} \frac{dS_{\text{source}}}{dg_{\mu\nu}}
\]

---
Back-reaction vs de Sitter no-go

• Few honest
  • work in 4D effective theory
  • work with moving branes in static backgrounds

• What do extra dimensions do during inflation?
  • Explicit example from 6D
    supergravity
  • cod 2

This term often vanishes

This term has definite (AdS) sign

This term is often omitted

then
\[ \frac{1}{2\kappa^2} \int R = S_{on-shell} + \int \nabla^2 e^{dW} + g_{\mu\nu} \frac{dS_{source}}{dg_{\mu\nu}} \]
Back-reaction vs de Sitter no-go

CB, Maharana, van Nierop, Nizami & Quevedo

Source-brane matching conditions imply these terms exactly cancel one another for codim-2 sources

\[
\frac{1}{2\kappa^2} \int R = S_{\text{on-shell}} + \int \nabla^2 \phi \, dW + g_{\mu\nu} \frac{dS_{\text{source}}}{dg_{\mu\nu}}
\]
Source-brane matching conditions imply these terms exactly cancel one another for codim-2 sources:

\[ \frac{1}{2\kappa^2} \int R = S_{on-shell} + \int \nabla^2 e^{dW} + g_{\mu\nu} \frac{dS_{source}}{dg_{\mu\nu}} \]

This term is generically a total derivative for higher-dim supergravities: \( S_{on-shell} = \int d\Omega \)
Motivation

This can have de Sitter sign

For example, this is a reason why the de Sitter compactifications of 6D sugra evade the no-go theorems

\[
\frac{1}{2\kappa^2} \int R = S_{\text{on-shell}} + \int \nabla^2 e^{dW} + g_{\mu\nu} \frac{dS_{\text{source}}}{d g_{\mu\nu}}
\]
Motivation

Because it is a total derivative it does not depend on most of the details of back-reacted solutions.

\[
\frac{1}{2\kappa^2} \int R = S_{on-shell} + \int \nabla^2 e^dW + g_{\mu\nu} \frac{dS_{source}}{dg_{\mu\nu}}
\]

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Extra-dimensional Inflationary Models

hep-th/0608083 w Tolley & de Rham
arXiv:1108.2553 w van Nierop
Higher-dimensional inflation

• What about time-dependent solutions?
  • Must generalize brane matching conditions to case where on-brane geometry is not maximally symmetric
  • Wish to solve higher-dimensional field equations exactly, including energetics of modulus stabilization
Higher-dimensional inflation

• What about time-dependent solutions?
  • Must generalize brane matching conditions to case where on-brane geometry is not maximally symmetric
  • Wish to solve higher-dimensional field equations exactly, including energetics of modulus stabilization

• For supersymmetric systems exact time-dependent scaling solutions are known
  • Can these be matched to sensible brane physics to see how brane properties control bulk fields?
Higher-dimensional inflation

- 6D Einstein-Maxwell-scalar system

$$ L = \frac{1}{2\kappa^2} [R + (\partial\phi)^2] + e^{-\phi} F_{mn} F^{mn} + C e^\phi $$

- Brane-localized inflaton, $\chi$

$$ L_{b1} = T_1 + e^{-\phi} \left[ (\partial \chi)^2 + V_1 e^{\lambda \chi} + \cdots \right] $$

$$ L_{b2} = T_2 $$
Higher-dimensional inflation

- Exact time-dependent solution
  \[ e^{-\phi} = (H_0\tau)^{c+2} e^{-\varphi(r)} \]

- FRW time in 4D Einstein frame
  \[ ds^2 = (H_0\tau)^c \left[ g_{mn} dx^m dx^n + \tau^2 (g_{ij} dx^i dx^j) \right] \]

\[ dt = \mp (H_0\tau)^{c+1} d\tau \]
Higher-dimensional inflation

- Exact time-dependent solution
  \[ e^{-\phi} = (H_0 \tau)^{c+2} e^{-\varphi(r)} \]
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- FRW time in 4D Einstein frame
  \[ dt = \mp (H_0 \tau)^{c+1} d\tau \]

- If \( c = -2 \) then \( a(t) = e^{H_0 t} \) and \( r \) constant

- 4D de Sitter geometry: evades no-go results due to near-brane asymptotics:
  \[ S_{on-shell} = \int \nabla^2 \phi \]
Higher-dimensional inflation

Tolley, CB, de Rham

- Exact time-dependent solution
  \[ e^{-\phi} = (H_0 \tau)^{c+2} e^{-\varphi(r)} \]

- FRW time in 4D Einstein frame
  \[ ds^2 = (H_0 \tau)^c [g_{mn} dx^m dx^n + \tau^2 (g_{ij} dx^i dx^j)] \]

- If \( c \neq -2 \) then \( a(t) = (H_0 t)^p \) and \( r(t) = (H_0 t)^{1/2} \)
  with \( p = (c + 1)/(c + 2) \)

\( \text{accelerated expansion if } p > 1 \text{ and so } c < -2 \)
Higher-dimensional inflation

- Source-bulk matching: how does it end?

- Add inflaton $\chi$ evolution to the equations

\[ L_b = T + e^{-\phi} \left[ (\partial \chi)^2 + V_0 + V_1 e^{\lambda \chi} + \cdots \right] \]

\[ \chi = \chi_0 + \chi_1 \ln(H_0 \tau) \]

Then \[ c + 2 = -\lambda \chi_1 \] controls the slow roll

and \[ H_0^2 = \lambda V_1 / [\chi_1 (3 + 2\lambda \chi_1)] \]
Higher-dimensional inflation

- Extra dimensions grow slowly as the noncompact four dimensions inflate
  - Inflation could help understand why extra dimensions have particular properties in our epoch
Higher-dimensional inflation

- Extra dimensions grow slowly as the noncompact four dimensions inflate
  - Inflation could help understand why extra dimensions have particular properties in our epoch
  - Evolving volume potentially allows the gravity scale to be high at horizon exit, but low at present (similar to Conlon, Kallosh, Linde & Quevedo)
Naturalness Issues


w van Nierop
An opportunity

• Brane backreaction provides a mechanism for lifting flat directions described by bulk moduli
  • Generalization of the Goldberger – Wise mechanism for RS codimension-1 sources
  • Can give exponentially large extra dimensions
An opportunity

- Brane backreaction provides a mechanism for lifting flat directions described by bulk moduli
  - Generalization of the Goldberger – Wise mechanism for RS codimension-1 sources
  - Can give exponentially large extra dimensions
- Brane backreaction cleanly identifies how curvature depends on brane properties
  - Backreaction competes with naïve tension terms
A puzzle

- For 6D flux-stabilized supergravity we have

\[
\frac{1}{2\kappa^2} \int R = S_{on-shell} = \frac{1}{2\kappa^2} \int \nabla^2 \phi \propto \frac{\delta S_b}{\delta \phi}
\]

and so \( R = 0 \) if no branes couple to 6D dilaton \( \phi \)

- Seems to imply geometry should be robustly flat, regardless of on-brane loops and perturbations
  - *This is superficially inconsistent with known perturbations of these geometries...*
A Simple Model

- 6D Einstein-Maxwell-scalar system

\[ L = \frac{1}{2\kappa^2} [R + (\partial\phi)^2] + e^{-\alpha\phi} F_{mn}F^{mn} + V(\phi) \]

- Two specific cases
  - 6D axion: \( \alpha = 0 \) and \( V = \Lambda \)
  - 6D supergravity: \( \alpha = 1 \) and \( V = \frac{2g_R^2}{\kappa^4} e^\phi \)
A Simple Model

- Simple solution

\[ ds^2 = \hat{g}_{mn} dx^m dx^n + [dr^2 + \alpha^2 L^2 \sin^2 \left( \frac{r}{L} \right) d\theta^2 ] e^{-\alpha \phi_0} \]

\[ F_{r\theta} = Q\alpha L \sin \left( \frac{r}{L} \right) e^{-\alpha \phi_0} \quad \phi = \phi_0 \]
A Simple Model

• Simple solution

\[ ds^2 = \hat{g}_{mn}dx^m dx^n + [dr^2 + \alpha^2 L^2 \sin^2 \left(\frac{r}{L}\right) d\theta^2]e^{-\alpha\phi_0} \]

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Magnetic flux required to stabilize extra dimensions against gravitational collapse

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A Simple Model

- Simple solution

\[ d s^2 = \hat{g}_{mn} d x^m \ d x^n + [d r^2 + \alpha^2 L^2 \sin^2 \left( \frac{r}{L} \right) d \theta^2] e^{-\alpha \phi_0} \]

\[ F_{r\theta} = Q \alpha L \sin \left( \frac{r}{L} \right) e^{-\alpha \phi_0} \quad \phi = \phi_0 \]

Labels flat direction (which exists due to shift symmetry or scale invariance)
A Simple Model

- Simple solution

\[ ds^2 = \hat{g}_{mn} dx^m dx^n + \left[ dr^2 + \alpha^2 L^2 \sin^2 \left( \frac{r}{L} \right) d\theta^2 \right] e^{-\alpha \phi_0} \]

\[ F_{r\theta} = Q\alpha L \sin \left( \frac{r}{L} \right) e^{-\alpha \phi_0} \quad \phi = \phi_0 \]

*For later:* notice radius is exponential in the flat direction \( \phi_0 \) in the SUSY case.
A Simple Model

- Simple solution (including back-reaction)

\[ ds^2 = \tilde{g}_{mn}dx^m \, dx^n + [dr^2 + \alpha^2 L^2 \sin^2 \left( \frac{r}{L} \right) d\theta^2] e^{-\alpha \phi_0} \]

\[ F_{r\theta} = Q\alpha L \sin \left( \frac{r}{L} \right) e^{-\alpha \phi_0} \quad \phi = \phi_0 \]

\[ 1 - \alpha = \frac{\kappa^2 T}{2\pi} \]
A Simple Model

• Simple solution (non-SUSY case)

\[ ds^2 = \hat{g}_{mn} dx^m \, dx^n + dr^2 + \alpha^2 L^2 \sin^2 \left( \frac{r}{L} \right) \, d\theta^2 \]

\[ F_{r\theta} = Q \alpha L \sin \left( \frac{r}{L} \right) \]

\[ \phi = \phi_0 \]

Field equations

\[ \frac{2}{L^2} = \kappa^2 \left( \frac{3Q^2}{2} + \Lambda \right) \]

\[ \hat{R} = \kappa^2 (Q^2 - 2\Lambda) \]

Flux quantization

\[ \frac{n}{g} = 2\alpha L^2 Q \]
A Simple Model

• Simple solution (non-SUSY case)

\[ ds^2 = \hat{g}_{mn} dx^m \, dx^n + dr^2 + \alpha^2 L^2 \sin^2 \left( \frac{r}{L} \right) \, d\theta^2 \]

\[ F_{r\theta} = Q \alpha L \sin \left( \frac{r}{L} \right) \]

\[ \phi = \phi_0 \]

\[ Q = \frac{n}{2\alpha g L^2} \]

\[ \hat{R} = \kappa^2 (Q^2 - 2\Lambda) \]

\[ \frac{1}{L^2} = \frac{8\alpha^2 g^2}{3n^2\kappa^2} \left[ 1 + \sqrt{1 - \left( \frac{3n^2\kappa^4\Lambda}{8\alpha^2 g^2} \right)} \right] \]
A Simple Model

- Simple solution (non-SUSY case)

\[ ds^2 = \hat{g}_{mn} dx^m dx^n + dr^2 + \alpha^2 L^2 \sin^2 \left( \frac{r}{L} \right) d\theta^2 \]

\[ F_{r\theta} = Q\alpha L \sin \left( \frac{r}{L} \right) \quad \phi = \phi_0 \]

Tune \( \Lambda = \frac{Q^2}{2} \) so \( \hat{R} = 0 \)

If \( T \to T + \delta T \) then \( \hat{R} \to -\frac{\kappa^2 \rho}{\pi \alpha L^2} \) where \( \rho = 2 \delta T \)
A Simple Model

• Simple solution (SUSY case)

\[ ds^2 = \hat{g}_{mn} \, dx^m \, dx^n + \left[ dr^2 + \alpha^2 L^2 \sin^2 \left( \frac{r}{L} \right) \, d\theta^2 \right] e^{-\phi_0} \]

\[ F_{r\theta} = Q\alpha L \sin \left( \frac{r}{L} \right) e^{-\phi_0} \quad \phi = \phi_0 \]

Field equations

\[ \frac{2g_R^2}{\kappa^2} = \frac{\kappa^2 Q^2}{2} \]

\[ \kappa^2 Q^2 L^2 = 1 \quad \hat{R} = 0 \]

Flux quantization

\[ \frac{n}{g} = 2\alpha L^2 Q = \frac{\alpha}{g_R} \]
The Puzzle

- In SUSY case, how does system respond to changes in brane tension?

Flux quantization: \( \frac{n}{g} = 2 \alpha L^2 Q = \frac{\alpha}{g_R} \)

Obstructs \( T \) to \( \delta T \)
The Puzzle

• In SUSY case, how does system respond to changes in brane tension?

\[ \frac{n}{g} = 2\alpha L^2 Q = \frac{\alpha}{g_R} \]

Flux quantization: Obstructs \( T \) to \( \partial T \)

• On other hand, general argument:

\[ \rho = \int dV \ L_{bulk} = -\frac{1}{2\kappa^2} \int dV \ \partial^2 \phi = \phi \ dS \ n \cdot \partial \phi \ \propto \ \frac{\partial T}{\partial \phi} \]
Puzzle Resolution

- Resolution: subdominant effects in the brane action are important for flux quantization

\[
L_b = T_b(\phi) + \Phi_b(\phi) * F + \ldots
\]
Puzzle Resolution

• Resolution: subdominant effects in the brane action are important for flux quantization

\[
L_b = T_b(\phi) + \Phi_b(\phi) \ast F + \ldots
\]

• New function \( \Phi \) has interpretation as brane-localized flux

\[
\frac{n}{g} = \int F + \frac{1}{2\pi} \sum_b \Phi_b e^\phi
\]
Puzzle Resolution

• Non-SUSY result when $\Phi$ nonzero:

$$V_{\text{eff}}(\phi) = \phi \int \frac{d\phi}{\phi^2} \left[ \frac{\pi \alpha L^2 \hat{R}(\phi)}{\kappa^2} \right]$$

Flat direction stabilized at:

$$\left[ \frac{\partial}{\partial \phi} \sum_b \delta T_b - Q \delta \Phi_b \right]_{\phi_*} = 0$$

Vac energy there:

$$\rho = \left[ \sum_b \delta T_b - 2Q \delta \Phi_b \right]_{\phi_*}$$
Puzzle Resolution

• SUSY result when $\Phi$ nonzero:

Flat direction stabilized at:

$$\left[ \delta T_b - 2Q\delta \Phi_b + \frac{1}{2} \frac{\partial}{\partial \phi} \sum_b \delta T_b - Q\delta \Phi_b \right]_{\phi_*} = 0$$

ie Einstein frame potential: $V = U(\phi)e^{2\phi}$
Puzzle Resolution

- **SUSY result when $\Phi$ nonzero:**

  Flat direction stabilized at:
  \[
  \rho = [\delta T_b - 2Q \delta \Phi_b] = \left[ -\frac{1}{2} \frac{\partial}{\partial \phi} \sum_b \delta T_b - Q \delta \Phi_b \right]_{\phi^*}
  \]

  Agrees with exact result for curvature

  Vac energy there:
  \[
  \rho = [\delta T_b - 2Q \delta \Phi_b] = \left[ -\frac{1}{2} \frac{\partial}{\partial \phi} \sum_b \delta T_b - Q \delta \Phi_b \right]_{\phi^*}
  \]
Applications

• Three intriguing choices:

Case 1: scale invariant (cf Weinberg’s no-go theorem):

if $\delta T$ independent of $\phi$ and $\delta \Phi = C e^{-\phi}$ then $V(\phi) = A e^{2\phi}$
Applications

- Three intriguing choices:

  Case 1: scale invariant:

  if $\delta T$ independent of $\phi$ and $\delta \Phi = Ce^{-\phi}$ then
  $V(\phi) = Ae^{2\phi}$

  Case 2: exponentially large volume:

  $\delta T_b = A + B (\phi + v)^2$ with $v \sim 50$ then
  $r = Le^{-\phi/2} \gg L$
Applications

- Three intriguing choices:

Case 3: parametrically small vacuum energy:

\[ \delta T_b \text{ and } \delta \Phi_b \text{ both independent of } \phi \text{ then } \rho = 0 \]

and \( \phi_* \) adjusts to satisfy flux quantization condition.
Applications

• Three intriguing choices:

Case 3: parametrically small vacuum energy:

\[ \delta T_b \text{ and } \delta \Phi_b \text{ both independent of } \phi \text{ then } \rho = 0 \]

and \( \phi_\ast \) adjusts to satisfy flux quantization condition

\[ \text{Brane action independent of } \phi \text{ stable against brane loops} \]
\[ \text{Bulk loops generate corrections of order } e^{2\Phi} = (1/r)^4 \]
Conclusions
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• Branes and brane back-reaction can have important implications for low-energy theory
  • Little explored beyond codimension 1
Conclusions

• Branes and brane back-reaction can have important implications for low-energy theory
  • Little explored beyond codimension 1

• Vast unexplored territory
  • Codim-2 back-reaction as big as brane effects
  • Promising for naturalness issues (different parametric dependences in energy; unusual stability to quantum corrections; etc)
Conclusions

• Branes and brane back-reaction can have important implications for low-energy theory
  • Little explored beyond codimension 1
• Vast unexplored territory
  • Codim-2 back-reaction
  • Promising for naturalness issues (different parametric dependences and unusual stability to quantum corrections, etc.)

Potentially wide-ranging observational implications for Dark Energy cosmology, the LHC and elsewhere…
Fin
Extra Slides
Calculation

• More general solutions

\[ ds^2 = e^{2W} \hat{g}_{mn} dx^m \, dx^n + dr^2 + e^{2B} d\theta^2 \]

\[ F_{r\theta} = Q e^{B-4W} \]

\[ \phi = \phi(r) \]
Calculation

• Perturb brane properties

\[ T \rightarrow T + \delta T(\phi) \]

• To evade time-dependence add current

\[ \Delta L_{\text{bulk}} = J\phi \quad \text{or} \quad \Delta L_{\text{bulk}} = J \]

• Find general solution to linearized equations

\[ \kappa^2 J L^2 \ll 1 \]
Calculation

• Sample solutions

\[ \delta W = W_0 + W_1 \cos \left( \frac{r}{L} \right) \]

\[ \delta \phi = \phi_0 + \phi_1 \ln \left( \frac{1 - \cos(r/L)}{\sin(r/L)} \right) - \kappa^2 J L^2 \ln \left[ \sin \left( \frac{r}{L} \right) \right] \]

and so on
Calculation

- Brane-bulk boundary conditions:

\[(e^B \phi')_b = \frac{\kappa^2}{2\pi} \left( \frac{\partial L_b}{\partial \phi} \right)\]

\[(e^B W')_b = \frac{\kappa^2}{4\pi} \left( \frac{\partial L_b}{\partial g_{\theta\theta}} \right) = U_b\]

\[(e^B B' - 1)_b = -\frac{\kappa^2}{2\pi} \left[ \left( \frac{\partial L_b}{\partial \phi} + \frac{3}{2} \frac{\partial L_b}{\partial g_{\theta\theta}} \right) \right]\]

Constraint: \[4U_b [2 - 2L_b - 3U_b] - \left( \frac{\partial L_b}{\partial \phi} \right)^2 = 0\]
Puzzle Resolution

- Non-SUSY result when $\Phi$ nonzero:

$$V_{eff}(\phi) = \phi \int \frac{d\phi}{\phi^2} \left[ \frac{\pi \alpha L^2 \hat{R}(\phi)}{\kappa^2} \right]$$

Flat direction stabilized at:

$$\left[ \frac{\partial}{\partial \phi} \sum_b \delta T_b - Q \delta \Phi_b \right]_{\phi_*} = 0$$

Vac energy there:

$$\rho = \left[ \sum_b \delta T_b - 2Q \delta \Phi_b \right]_{\phi_*}$$
Puzzle Resolution

• SUSY result when $\Phi$ nonzero:

Flat direction stabilized at:

$$\left[ \delta T_b - 2Q\delta \Phi_b + \frac{1}{2} \frac{\partial}{\partial \phi} \sum_b \delta T_b - Q\delta \Phi_b \right]_{\phi_*} = 0$$

ie Einstein frame potential: $V = U(\phi)e^{2\phi}$
Puzzle Resolution

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\[
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\]

Vac energy there:

\[
\rho = [\delta T_b - 2Q \delta \Phi_b] = \left[ -\frac{1}{2} \frac{\partial}{\partial \phi} \sum_b \delta T_b - Q \delta \Phi_b \right]_{\phi_*}
\]