Microstate Geometries and Non-BPS Black Objects

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Based upon work with I. Bena, N. Bobev, G. Dall'Agata, S. Giusto and C. Ruef
The Issue

Find and classify regular, horizonless solutions with the same asymptotic structure as a given black hole or black ring* ⇔ Microstate Geometries (definition)

Types of microstate geometries

- Introduction and background
- BPS/supersymmetric Extensively studied
- Extremal, non-BPS Recent work
**Motivation**  **Goal:** Better description of a stringy black hole ...

- Every semi-classical description involves some “loss of information”, or coarse graining ....
- GR + field theory in four dimensions: “extremely coarse grained” (uniqueness)
- How good is supergravity in 10 or 11 dimensions at “sampling” the states?
- Good enough for a semi-classical description of black-hole thermodynamics, e.g. entropy?

**Strominger and Vafa:** hep-th/9601029

\[ g_s = 0 \]

- **exremely coarse grained** singular, symmetric black-hole metric
- “suitably dense” family of representative regular less-symmetric metrics?
- A combination??
A better model

- Find: Vast number of smooth, horizonless “microstate geometries” in higher dimensional supergravity .... (to be discussed here)

- What states, and how many, are the smooth geometries capturing?
  - Smooth, horizonless ⇒ better (unitary) scattering properties
  - Smooth, horizonless geometries can be given a more precise interpretation using holography  
    e.g. Skenderis and Taylor
  - Smooth, horizonless ⇒ a better description of the physics of the IR limit of the dual field theory

- Are microstate geometries typical? Do they come from the sector or the dual field theory that contributes most to the entropy?

- Even if you find compelling physical reasons for preferring a more symmetric, singular solution over a less symmetric regular solution what are the “other sectors” represented by the regular solutions ....
Are all microstate geometries simply Planck-scale fuzz?

★ The potential danger: As $g_s$ increases, gravity gets stronger ⇒ Matter and presumably the microstate structure get compressed and hence grow smaller

.... but horizon size grows with $g_s$.  

The Black-Hole Correspondence Principle (Horowitz and Polchinski)

★ To get macroscopic, horizon sized microstate geometries:

Microstate structure must grow with $g_s$

• How can this be arranged?
• Generic resolutions are horizon sized?
Key Components

★ Singularity resolutions through geometric transitions: Replace singular brane source with smooth (topological) fluxes

★ Many Low mass modes/excitations of smooth geometry
  ⇒ • Large-scale geometric effects
  ⇒ • Lots of microstates

★ Microstate geometries grow in size with $g_s$ at exactly the same rate as the size of a black hole.

  $D$-brane tension $\sim g_s^{-1} +$ angular momentum
  ⇒ Branes spread out with increasing $g_s$
  ⇒ Microstate geometries typically extend to horizon scale

★ Magnetic fluxes: Regularity and Low mass modes
Branes “expanded” and smoothed out by dipolar fluxes

Two-charge “microstate geometries”

- Mathur et al.; Lunin, Maldacena, Maoz

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**D1-D5 branes**

![D1-D5 branes](image)

**D1-D5 branes + KKM charge + angular momentum**

![D1-D5 branes + KKM charge + angular momentum](image)

Low mass modes: The supertube profile in $R^4$

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**Resolution of timelike singularity**

Black hole

**singular metric**

**smooth metric**

Microstate geometry
**Problems:** Two charge horizon areas are still very close to Planck scale ....

“Small Black Holes:” Microstate geometries size dependence on $g_s$ depends on duality frame.

**To do better:** Need to study *three-charge* black holes in five dimensions

- BPS three-charge black holes have macroscopic horizons
- Size of a three-charge supertube grows with $g_s$ [Bena and Kraus, hep-th/0402144]
Three-charge BPS black holes in five dimensions

★ Metric

\[ ds_5^2 = -\left(Z_1 Z_2 Z_3\right)^{-2/3} (dt + k)^2 + \left(Z_1 Z_2 Z_3\right)^{1/3} ds_4^2 \]

★ Electromagnetic fields

\[ F^{(I)} = d\left(Z_I^{-1} (dt + k)\right) + \Theta^{(I)} \]

- Spatial base = \(\mathbb{R}^4\) and \(\Theta^{(I)} = 0\) ⇒ Singular, point sources for harmonic \(Z_I\)

⇒ Three-charge BMPV black hole

\[ S = 2\pi \sqrt{Q_1 Q_2 Q_3 - J^2} \]

- Spatial base = \(\mathbb{R}^4\) and \(\Theta^{(I)} \neq 0\) ⇒ Singular line sources for \(\Theta^{(I)}\) and \(Z_I\)

⇒ Three-charge BPS black rings

- \(\Theta^{(I)}\) determine the ring profile; \(Z_I\) charge distributions along the ring

BPS: Mass = electric charge

M-theory: M2 branes

Magnetic dipoles

M-theory: M5 branes
The BPS Equations: A four step process

(0) Pick a hyper-Kähler four-dimensional, spatial metric for the base \((ds_4)\) (with self-dual curvature) asymptotic to \(R^4\)

On this base, solve the following equations, in the following order:

(1) **Magnetic**

\[ \Theta^{(I)} = \star_4 \Theta^{(I)} \]

Black ring sources/data
Freely choosable M5 brane ring profile: \(\vec{F}(v)\)

(2) **Electric** (sourced by magnetic)

\[ \nabla^2 Z_I = \frac{1}{2} C_{IJK} \star_4 (\Theta^{(J)} \wedge \Theta^{(K)}) \]

\[ C_{IJK} \equiv |\varepsilon_{IJK}| \]

+ M2 brane densities, \(\rho_K(v)\), along ring profile

(3) **Angular momentum**

\[ dk + \star_4 dk = Z_I \Theta^{(I)} \]

Adjust homogeneous solutions to avoid closed time-like curves

Originally found in work on five-dimensional supergravity

*Gauntlett, Gutowski, Hull, Pakis and Reall*, hep-th/0209114; *Gauntlett and Reall*, hep-th/0401129

*Bena and Warner*, hep-th/0408106

A linear system
Generalized “Supertubes”: Zero-Horizon-Area Black Rings

$\bar{F}(v) = \text{magnetic profile}$

$\rho_I(v) = \text{M2 brane densities along the ring profile}$

$A(v) = \text{Area profile function}$

(Generalized) Supertubes: $A(v) = 0 \Rightarrow$ Six arbitrary functions

★ Size of a three-charge supertube grows with $g_s$ exactly like that of a black hole horizon.

Null orbifold singularity:

Singularity resolved by a new geometric transition in the non-compact hyper-Kähler base space geometry...
The Geometric Transition  \[ \text{Branes} \rightarrow \text{Cohomological Fluxes} \]

Microstate geometries must have the same supersymmetries as the original “classical” black hole or black ring

\[ \Rightarrow \quad ds_4^2 \text{ can be any (self-dual) hyper-Kähler metric} \]

\[ \Rightarrow \quad \text{Lots of cohomology} \quad \Rightarrow \quad \text{Lots of interesting fluxes} \]

**Problem:** The only regular, Euclidean, hyper-Kähler metric that is asymptotic to flat \( \mathbb{R}^4 \) is the globally flat Euclidean metric on \( \mathbb{R}^4 \)

\[ \Rightarrow \quad \text{No cohomology} \quad \Rightarrow \quad \text{Singular sources are the only option} \]

**Resolution:** \textbf{Ambi-polar bases}

The complete five-dimensional metric needs to be Lorentzian:

\[ ds_5^2 = -Z^{-2}(dt + k)^2 + Z ds_4^2 \]

\[ \Rightarrow \quad \text{the four-dimensional base metric can be ambi-polar:} \]

It can change its overall sign ... provided \( Z \) changes sign at same place

\((\text{Generic consequence of equations of motion??})\)

• Huge variety of newly admissible, hyper-Kähler bases asymptotic to \( \mathbb{R}^4 \)

\[ \Rightarrow \quad \text{Lots of cohomology for smooth resolutions} \]
Invaluable Example: Gibbons-Hawking metrics (hyperKähler)

\[ ds_4^2 = V^{-1} (d\psi + A)^2 + V \, d\vec{y} \cdot d\vec{y} \quad \vec{y} \in \mathbb{R}^3 \quad V = \sum_{j=1}^{N} \frac{q_j}{|\vec{y} - \vec{y}(j)|} \quad \vec{\nabla} \times \vec{A} = \vec{\nabla} V \]

- Regularity at \( y = y^{(i)} \) \( \Rightarrow \) \( q_j \) is a integer

- Asymptotic to \( \mathbb{R}^4 \) \( \Rightarrow \) \( V \sim r^{-1} \) \( \Rightarrow \) \( \sum_j q_j = 1 \)

Ambi-polar bases \( \Rightarrow \) Lots of possibilities + Lots of 2-cycles

Non-trivial 2-cycles, \( \Delta_{ij} \): Any curve from \( y^{(i)} \) to \( y^{(i)} \) and \( U(1) \) fiber (\( \Psi \)) above it

Fiber collapses to a point when \( y = y^{(i)} \)

Two intersecting 2-spheres: \( \Delta_{ij} \) and \( \Delta_{jk} \)
The Geometric Transition: Blowing \textit{bubbles}

Flat R\(^4\): \[ ds_4^2 = V^{-1}(d\psi + A)^2 + V \left( dr^2 + r^2(d\theta^2 + \sin^2 \theta \, d\phi^2) \right) \]

\[ V = \frac{+1}{r} \]

Pair-create new geometric charges:

\[ \implies \text{Blow bubbles} \]

\[ V = \frac{1}{r} + \frac{q}{r_1} - \frac{q}{r_2} \]

\[ r_j \equiv |\vec{y} - \vec{y}^{(j)}| \]

\[ V = \frac{1}{r} + \frac{q_1}{r_1} - \frac{q_1}{r_2} + \frac{q_2}{r_3} - \frac{q_2}{r_4} \]
The “Magnetic Flux” \( \Theta^{(I)} = \star_4 \Theta^{(I)} \)

**Geometric Transition:** Look for solutions with *no singular sources* (no branes):

All fluxes are topological, supported by the cycles, \( \Delta_{ij} \).

Potential:

\[
V = \sum_{j=1}^{N} q_j r_j, \quad \sum_{j=1}^{N} q_j = 1, \quad q_j \in \mathbb{Z} \quad r_j \equiv |\vec{y} - \vec{y}^{(j)}|
\]

Fluxes determined by smooth functions, \( V^{-1} K^I \), with \( K^I = \sum_{j=1}^{N} \frac{k^I_j}{r_j} \).

Topological fluxes through cycle, \( \Delta_{ij} \):

\[
\Pi^{(I)}_{ij} = \left( \frac{k^I_j}{q_j} - \frac{k^I_i}{q_i} \right)
\]

**The Warp Factors:**

\[
\nabla^2 Z_I = \frac{1}{2} C_{IJK} \star_4 (\Theta^{(J)} \wedge \Theta^{(K)})
\]

\[
Z_I = 1 + \frac{1}{2} C_{IJK} V^{-1} K^J K^K + L_I \quad \nabla^2 y L_I = 0
\]

No singular sources:

\[
L_I = -\frac{1}{2} C_{IJK} \sum_{j=1}^{N} \frac{k^I_j k^K_j}{q_j r_j} \quad \text{Cancels the divergences as } r_j \to 0
\]

Similarly for the angular momentum vector, \( k \)

**Given the fluxes, the solution is fixed by the absence of singular sources**
Bubbled Geometries

Think of the GH points as locations of D6 branes

\[ \text{BPS} + \text{anti-BPS} \Rightarrow \]
\[ \text{Net attraction} \]

\[ \text{Bubbles} + \text{Flux} \Rightarrow \]
\[ \text{Expansion force} \]

\[ \Rightarrow \text{Equilibrium BPS Configuration} \]

Size of bubble =
\[ \text{Separation of GH points when attraction balances fluxes expansion} \]

- Fluxes fix N lengths, “L”
- 2N moduli: \( \theta, \phi \) ...
  \textit{low-mass excitations}
Some features

★ These are smooth resolutions of BPS black hole singularities, capping off the solution at the horizon scale ... 

★ Microstate geometries are macroscopic and grow with $g_s$ at exactly the same rate as the size of the black hole horizon.

★ There are Deep/Scaling microstate geometries with throats that can be made arbitrarily long but cap off smoothly. Exactly like a black hole up to its (macroscopic) horizon

★ All these Deep/Scaling solutions represent microstates (using holography in the long AdS throat)

★ Analysis of the mass gap of excitations of the deep/scaling microstate geometries imply they are representatives of the “typical microstates” that provide the dominant contribution to the entropy ...

$$E_{gap} \sim (C_{cft})^{-1}$$

de Boer, El-Showk, Messamah, Van den Bleeken, arXiv:0807
Entropy

★ Discrete choices: Partitioning fluxes amongst the bubbles
★ Continuous moduli:
  Orientations/angles
  between bubbles

Semi-classical contributions to entropy:

**Crude estimates**  Bena, Wang and Warner, arXiv hep-th/0604110

Topological Entropy from partitioning fluxes over bubbles:  \( \sim Q^{1\frac{1}{4}} \)

Positions of bubbles in five dimensions:  \( \sim Q^{1\frac{1}{2}} \)

Black hole entropy:  \( S \sim \sqrt{Q_1 Q_2 Q_3} \sim Q^{3\frac{1}{2}} \)

Careful semi-classical quantization and extensive analysis, including all small fluctuations around such backgrounds. Same conclusion:

**Not enough semi-classical entropy**

Going Far Beyond the Transition: Fluctuating Geometries

The problem: **Not Enough low mass modes**

- Gibbons-Hawking bases still have **too much symmetry**: \((S^1)^7\)
- “Graviton gas” of fluctuations about such backgrounds also insufficient

**More low mass modes:** Allow the shapes of bubbles to undergo large (solitonic) fluctuations as functions of compactified (internal) directions ...

**⇒ Bubbled geometries** with shapes that depend upon **arbitrary functions**

Simplest first step: Put fluctuating supertubes in scaling bubbled backgrounds

\[
ds^2_{\text{IIB}} = Z_1^{-1/2}Z_2^{-1/2} \left( -(dt + k)^2 + (dz + B)^2 \right) + Z_1^{1/2}Z_2^{1/2} ds_4^2 + Z_1^{1/2}Z_2^{-1/2} ds_{T4}^2
\]

six-dimensional geometry

**⇒** Look for backgrounds that have dependence on one of the internal coordinates, \(z\). Bubbles have shapes that fluctuate as a function of \(z\).
Entropy Enhancement

Expect entropy like that of a supertube

\[ S \sim \sqrt{Q_1 Q_2} \sim Q \]

Actual Entropy of fluctuations

\[ S \sim \sqrt{Q_1^{\text{effective}} Q_2^{\text{effective}}} \sim Q^{\text{effective}} \]

Recall the Warp factors:

\[ Z_I = 1 + \frac{1}{2} C_{IJK} V^{-1} K^J K^K + L_I \]

Asymptotic charges:

\[ Z_I \sim 1 + \frac{Q_I}{r} \quad \text{as} \quad r \to \infty \]

Effective charges:

\[ Z_I \sim \frac{Q_I^{\text{effective}}}{r_*} \quad \text{as} \quad r_* \to 0 \]

Charge from infinity

\[ r_* = 0 \]

Charge near tube

Magnetic dipole contributions can become vast in deep scaling solutions near critical (\( V=0 \)) surfaces

\[ Q_I^{\text{effective}} \gg Q_I \]

New physical effect arising out of strong magnetic dipole interactions
Supertube angular momentum limit (absence of CTC’s):

\[ |J| \leq \sqrt{Q_1 \text{ effective} Q_2 \text{ effective}} \sim Q_{\text{effective}} \]

Supertube can have vast \( J \Rightarrow \text{very large and floppy} \), with \text{large number of modes}.

★ Not a red-shift effect ★ Not a tension renormalization

Requires \textit{magnetic dipoles} and \textbf{very strong} near \textit{critical (V=0) surfaces} in \textit{Deep/Scaling solutions}

Recent results Bena, Bobev, Giusto, Ruef and Warner, \texttt{arXiv:1006.3497}

★ Entropy enhancement was initially based upon brane probes but we now have \textbf{fully back-reacted solutions that show Entropy Enhancement can be realized exactly}.

• Limits on supertube entropy determined by other (fixed) charges

• Multiple “wiggling supertube” enhancement feedback?

• Limits on enhancement for suitably generic solutions?

\[ S \sim \sqrt{Q_1 \text{ effective} Q_2 \text{ effective}} \sim Q_{\text{effective}} \]

\[ Q_{\text{effective}} \sim (Q_{\text{asymptotic}})^{\frac{3}{2}} \]

• New class of enhanced “wiggling” solutions goes beyond estimates and semi-classical counting to date
Message:

- Bubbled geometries with deep, scaling throats look just like black holes but cap off smoothly without a horizon.
- Scaling geometries are representatives in the **typical sector** of the theory.
- Vast majority of microstates associated with "bubble shape modes" that depend upon internal dimensions, and localize near "critical surfaces".
- We still have not yet shown that semi-classical fluctuations have $S \sim Q^{3/2}$ but we can do better than $S \sim Q^1$ via entropy enhancement.

Work in progress

- More general classes of bubble shape modes: "Double Bubbling"

Supertubes with 3 charges and 2 dipole charges ⇒
Shapes as functions of two variables.

![Diagram showing the transition from pointlike to profile functions to shape functions](image)
The Scheme of Things: The Story So Far

$g_s = 0$

**Multi-center Quiver Quantum Mechanics**
Denef - Moore (2007)

$\mathcal{S}_{\text{CFT}} \sim \mathcal{S}_{\text{BH}}$

**Scaling solutions**

**Black-hole deconstruction**
Denef, Gaiotto, Strominger, Van den Bleeken, Yin (2007)

$\mathcal{S}_{\text{CFT}} \sim \mathcal{S}_{\text{BH}}$

**Critical Surfaces**
$V = 0 \Rightarrow$ Entropy

**Singular sources**

$g_s$ increasing
can resolve and grow
No singularity, no horizon

Typicality + Black-hole-like:
Scaling microstate geometries

Semi-classical shape modes depending on internal directions near Critical Surfaces

Representative sample?
Extremal, Non-BPS Solutions

• Examples of microscopic state counting
  e.g. Horowitz and Emparan, hep-th 0607023

• The singularity is still time-like ... the Penrose diagram is still the same: why should its resolution be any different from the BPS story?

• First step: Construct families of multi-centered extremal, non-BPS black holes and black rings ...

• Look for smooth, horizonless analogs

**Problem:** Now have to cope with full, second order, non-linear Einstein equations. **Hard!** Particularly for solutions involving more than one variable

**Morally:** *Extremal* should be easier.
A Recent (Somewhat) Systematic Physical Approach

Bena, Dall’Agata, Giusto, Ruef and Warner, arXiv:0902.4526

• **Locally BPS** or **Almost BPS** formulations
  ★ Make solutions out of supersymmetric elements that “disagree” about the supersymmetry.
  ★ Angular momentum plays a crucial role in stabilization
  ★ *Supersymmetry breaking* controlled by the separation of the *locally supersymmetric* pieces

• Floating Brane Ansatz
  ★ *Extremal*: There should be brane probes that feel no force
  ★ Make an Ansatz based upon such a “floating brane” probe
    
    Relates *Maxwell* fields to *metric* coefficients

  ★ Look for simple (*linear?*) systems that solve that Ansatz
“Almost BPS” Solutions

(0) Hyper-Kähler four-dimensional, spatial metric asymptotic to $\mathbb{R}^4$ or $\mathbb{R}^3 \times S^1$

(1) $\Theta^{(I)} = \star_4 \Theta^{(I)}$

(2) $\nabla^2 Z_I = \frac{1}{2} C_{JK} \star_4 (\Theta^{(J)} \wedge \Theta^{(K)})$

(3) $dk + \star_4 dk = Z_I \Theta^{(I)}$

**BPS**: Base must have **self-dual curvature** to be consistent with the supersymmetries of the other charges: M2-M2-M2 or D2-D2-D2 + D6 in IIA

**Almost-BPS**: Solve the same **linear** equations for a base with **anti-self-dual curvature**: (D2-D2-D2) + **anti-D6**. Solves equations of motion.

Generalized to generate a huge number of multi-centered, non-BPS solutions in five dimensions (base asymptotic to $\mathbb{R}^4$) and four dimensions (base asymptotic to $\mathbb{R}^3 \times S^1$ - Taub-NUT)

Bena, Dall’Agata, Giusto, Ruef and Warner, arXiv:0902.4526
Bena, Giusto, Ruef and Warner, arXiv:0908.2121
Results I

This simple trick generates a substantial fraction of the known extremal, non-BPS solutions and a large number of new, more general solutions ......

- For example: the most general extremal non-BPS under-rotating black hole in four dimensions.
  Rasheed-Larsen black hole: Two charges, D0-D6
  Dualized + KKM charge

Almost-BPS technique: 5 fully independent charges, D0-D2-D2-D2-D6

Bena, Dall’Agata, Giusto, Ruef and Warner, arXiv:0902.4526

Important because this is exactly enough charges to “seed” the most general solution via $E_{7(7)}$ action on the charges

- Solutions are generically much more complicated than BPS counterparts
  Built out of BPS bits, can assemble 75% of the solution by transforming BPS components ... the hard work lies in the rest of it, but still linear

- Non-BPS features and complexity emerge in global properties:
  (i) More complicated sources for the linear equations
  (ii) Regularity, like absence of CTC’s. (iii) Asymptotic charge structure
Results II: Multi-centered solutions

Multi-centered solutions lie at the heart of recent developments for BPS systems: Our understanding of black-hole microstates, entropy enigmas, attractor flows, black-hole deconstruction, wall-crossing of black hole indices .... Denef’s talk

Non-linear Einstein equations are hard, particularly for solutions involving more than one variable ...

⇒ Multi-centered, non-BPS systems were considered very difficult

Linear system ⇒ combining solutions is trivial because of superposition
⇒ explicit construction of non-BPS multi-black-hole/multi-black-ring solutions

Bena, Giusto, Ruef and Warner, arXiv:0908.2121

“Almost-BPS” technology has been remarkably successful in constructing more general, new families of non-BPS black holes and black rings

⇒ Can take interesting BPS results and study non-BPS counterparts

• Supersymmetry breaking scale set by separation of rings and black holes

Problem: Bubbling using “Almost BPS” is hard. Cohomological self-dual fluxes in an anti-self-dual background metric are non-normalizable.
More General Linear Systems: The Floating Brane Ansatz


- Choose a species of probe
- Make an Ansatz so that it floats

Relates Maxwell fields to metric

\[ ds^2_5 = - (Z_1 Z_2 Z_3)^{-2/3} (dt + k)^2 + (Z_1 Z_2 Z_3)^{1/3} ds^2_4 \]
\[ F^{(I)} = d(Z^{-1}_I (dt + k)) + \Theta^{(I)} \]

- Find and solve linear sub-systems of the equations of motion

Simplest linear sub-system:

\[ (1,2,3) \quad \Theta^{(I)} = \star_4 \Theta^{(I)} \quad \nabla^2 Z_I = \frac{1}{2} C_{IJK} \star_4 (\Theta^{(J)} \wedge \Theta^{(K)}) \quad dk + \star_4 dk = Z_I \Theta^{(I)} \]

\[ (0) \quad \textbf{Ricci flat} \text{ four-dimensional, spatial metric asymptotic to } \mathbb{R}^4 \text{ or } \mathbb{R}^3 \times S^1 \]

E.g. Euclidean Schwarzschild, Kerr-Taub-Bolt

⇒ Smooth, bubbled solutions: The “bolt” is the bubble

Euclidean Kerr-Taub-Bolt: Extended ranges of allowed parameters because base can be to be ambi-polar
More General Linear Sub-systems: Electrovac Base Metrics

*(Floating M2 branes in three-charge background)*

(0) **Base metric can be any Euclidean Einstein-Maxwell solution**

Choose

\[
R_{\mu\nu} = \frac{1}{2} \left( F_{\mu\rho} F^{\rho}_{\nu} - \frac{1}{4} g_{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} \right)
\]

Decompose:

\[
F = \Theta^{(3)} - \omega^{(3)} \quad \Theta^{(3)} = *\Theta^{(3)} , \quad \omega^{(3)} = - * \omega^{(3)}
\]

(1) Solve for \( Z_1, \Theta^{(2)}, Z_2 \) and \( \Theta^{(1)} \)

\[
\nabla^2 Z_1 = * (\Theta^{(2)} \wedge \Theta^{(3)}) , \quad (\Theta^{(2)} - *\Theta^{(2)}) = 2Z_1 \omega^{(3)}
\]

\[
\nabla^2 Z_2 = * (\Theta^{(1)} \wedge \Theta^{(3)}) , \quad (\Theta^{(1)} - *\Theta^{(1)}) = 2Z_2 \omega^{(3)}
\]

(2) Solve for \( Z_3 \) and \( k \)

\[
\nabla^2 Z_3 = * \left[ \Theta^{(1)} \wedge \Theta^{(2)} - \omega^{(3)} \wedge (dk - *4dk) \right]
\]

\[
dk + *dk = \frac{1}{2} \sum_{I=1}^{3} Z_I (\Theta^{(I)} + *\Theta^{I})
\]

Equations (1,2,3) now more complicated but still a coupled linear system:
Example: Israel-Wilson metrics

\[ ds^2_4 = (V_+ V_-)^{-1}(d\psi + \vec{A} \cdot d\vec{y})^2 + (V_+ V_-)(dy_1^2 + dy_2^2 + dy_3^2) \]

\[ \vec{\nabla} \times \vec{A} = V_+ \vec{\nabla}V_+ - V_- \vec{\nabla}V_- \]

\[ \Theta^{(3)} \leftrightarrow V_+ \quad \omega_- \leftrightarrow V_- \]

Can explicitly find solution for D0-D2-D4-D6 black hole in anti-D6 background

The D6 and anti-D6 charges are held apart by normalizable fluxes through the corresponding cycles ⇒ smooth, bubbled non-BPS solutions with fluxes

Another example: Euclidean Kerr-Newman base with electric, magnetic and NUT charges

Bobev and Ruef, arXiv:0912.0010

⇒ Smooth, “dyonic bolt” solution
Floating brane probe is actually “whirling” around the internal (compactified) dimensions with some constant velocity, $v$ ....

- New classes of solution, including those obtained by boosting and dualizing. Solutions have new parameters (components of $v$)
- Includes: Non-Supersymmetric Black Rings Elvang, Emparan and Figueras
  Non-Supersymmetric Bubbled Geometries Jejjala, Madden, Ross and Titchener
  Over-rotating Rasheed-Larsen
Extremal Non-BPS Developments

Four-dimensional base

- Ricci-flat
  - hyper-Kähler
  - whirling, floating branes

- Electro-vac
  - More!
  - Others
    - Israel-Wilson
    - Others

- Bolts; Others
  - Dyonic bolts

BPS
- Some almost BPS
- Almost BPS
Added Bonus:

These techniques generate some interesting examples of non-extremal smooth, bubbled solutions

• E.g. From a Euclidean Schwarzschild base:

\[ M_0 = \frac{\pi}{4G_5} \left( 16m^2 + \frac{1}{4\pi^2}(Q^1 + Q^2 + Q^3) \right) \]

Smooth (microstate) geometry: the bolt is the bubble

Schwarzschild “mass,” \( m \), is not a conserved charge but contributes to the total mass

The bottom line

There are evidently a huge variety of new “extremal” black-hole solutions and bubbled solutions that can be obtained in this way.

Can generate most known solutions and significantly extend families of solutions

Very optimistic that microstate geometries can be extended to extremal black holes to give (i) smooth geometries (ii) many low mass modes
Conclusions

★ One can resolve the time-like singularities of BPS solutions in terms of smooth, horizonless microstate geometries

★ Microstate geometries have vast numbers of low mass modes that make the resolutions “large” (horizon sized) with respect to the Planck scale
  - Scaling geometries: Representatives of typical sector

★ Better description of a stringy BPS black hole, particularly when combined with insights of holographic field theory

★ Dominant contribution to the number of states come from “bubble shape modes” that depend upon internal dimensions, and localize near “critical surfaces” in the resolution
  - Magnetic dipole interactions crucial to entropy enhancement
  - Convergence with quiver quantum mechanics at small gs

★ “Almost BPS” and “Floating Brane” techniques are very powerful tools in obtaining new families of non-BPS solutions

★ Very probable that whatever successes come from the study of BPS solutions can be taken over to extremal solutions....

★ Likely that the features of BPS microstate geometries will also persist in near BPS solutions....