Neutrino cross sections: from 5 GeV to 50 GeV

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Recent MINOS measurements

MINOS, Adamson et al, PRD81, 072002 (2010)
Plan

- Start with E=50 GeV and move down in energy
  - Kinematic corrections
  - Physics processes: DIS, exclusive contributions
  - Target mass corrections (TMC)
  - Even with DIS, non-perturbative extrapolations of structure functions
CC interactions

\[
\frac{d^2 \sigma^{\nu(\bar{\nu})}}{dx \, dy} = \frac{G_F^2 M_N E_\nu}{\pi (1 + Q^2/M_W^2)^2} \left\{ y^2 x F_1^{W^\pm} + (1 - y(1 + \frac{M_N x}{2E_\nu})) F_2^{W^\pm} \pm (xy(1 - \frac{y}{2}) F_3^{W^\pm} \right\}
\]

The usual variables:

\[
q^2 = (k - k')^2 = -Q^2
\]

\[
x = \frac{Q^2}{2P \cdot q}
\]

\[
y = \frac{P \cdot q}{P \cdot k}
\]
Lepton mass corrections-CC interactions

\[
\frac{d^2\sigma^{\nu(\bar{\nu})}}{dx\,dy} = \frac{G_F^2 M_N E_\nu}{\pi(1 + Q^2 / M_W^2)^2} \left\{ \left( y^2 x + \frac{m_\tau^2 y}{2E_\nu M_N} \right) F_1^{W\pm} \right. \\
+ \left[ (1 - \frac{m_\tau^2}{4E_\nu^2}) - (1 + \frac{M_N x}{2E_\nu}) y \right] F_2^{W\pm} \\
\pm \left[ xy(1 - \frac{y}{2}) - \frac{m_\tau^2 y}{4E_\nu M_N} \right] F_3^{W\pm} \\
+ \frac{m_\tau^2 (m_\tau^2 + Q^2)}{4E_\nu^2 M_N^2 x} F_4^{W\pm} - \frac{m_\tau^2}{E_\nu M_N} F_5^{W\pm} \right\} .
\]

Albright-Jarlskog

\[ F_4 = 0 \]
\[ F_5 = F_2 / (2x) \]

Mass corrections in the matrix element squared.

Kinematic limits also modified – x and y.

50 GeV comparison of tau neutrino and muon neutrino CC cross sections

$\nu_\tau N$, $\bar{\nu}_\tau N$ reduced by about 35-40% relative to muon neutrino/antineutrino cross sections

Solid – neutrinos
Dashed - antineutrinos
Tau mass corrections in neutrino-N scattering, another plot over broader range

Figure from Paschos and Yu, Phys. Rev. D 65 (2002)
Components of the Cross Section

Lipari, Lusignoli and Sartogo, PRL 74 (1994)

Need to separate out the different contributions:

\[ W > W_{\text{min}} \quad W^2 = (p + q)^2 \]

- Deep inelastic scattering
- Quasi-elastic scattering
- Single pion production

Typical separation of contributions.

Previous formula is the DIS term.

Qel and pion production can be approx. 25% of the cross section at 5 GeV.

\[ \nu_\mu \langle N \rangle \]

Figure 1

NuMI

Lipari, Lusignoli and Sartogo, PRL 74 (1994)
Quasi-elastic scattering

Fig. from Kuzmin, Lyubuskin & Naumov, Phys. Atom. Nucl. 69 (2006)

\[ \sigma \text{ NOT } \sigma/E \]

Divide by 2 for \( \nu N \).
Pion production (exclusive)

Fig. from Kuzmin, Lyubuskin & Naumov, Phys. Atom. Nucl. 69 (2006)

$\sigma^{tot}/E_{\nu}, \text{10}^{-38} \text{ cm}^2/\text{GeV}$

$\nu_\mu N$

$\chi^2/\text{NDF} = 892/664 = 1.34$

5 GeV

DIS-Target mass corrections at low energies

- Light cone moment fraction: Bjorken $x$ replaced by Nachtman variable $\xi$, $\eta$
- Target mass corrections to structure functions
- Mass corrections in the cross section formula

\[
\frac{1}{\xi} = \frac{1}{2x} + \sqrt{\frac{1}{4x^2} + \frac{M^2}{Q^2}}
\]

\[
\xi = \frac{2x}{1 + \sqrt{1 + \frac{4M^2x^2}{Q^2}}}
\]

Graph showing the relationship between $\xi$, $\eta$, and $x$. The graph includes a line labeled $Q^2 = 1$ GeV$^2$. The equation is plotted on the graph with the given parameters.
Results of TMC

\[
F_{1}^{TMC}(x, Q^2) = \frac{x}{\xi \rho} F_{1}^{(0)}(\xi) + \frac{M^2 x^2}{Q^2 \rho^2} h_2(\xi) + \frac{2M^4 x^3}{Q^4 \rho^3} g_2(\xi)
\]

\[
F_{2}^{TMC}(x, Q^2) = \frac{x^2}{\xi^2 \rho^3} F_{2}^{(0)}(\xi) + \frac{6M^2 x^3}{Q^2 \rho^4} h_2(\xi) + \frac{12M^4 x^4}{Q^4 \rho^5} g_2(\xi)
\]

\[
F_{3}^{TMC}(x, Q^2) = \frac{x}{\xi \rho^2} F_{3}^{(0)}(\xi) + \frac{2M^2 x^2}{Q^2 \rho^3} h_3(\xi) + 0
\]

\[
F_{j}^{(0)} \equiv \left( \lim_{M \to 0} F_{j}^{TMC}(x, Q^2) \right) \bigg|_{x=\xi}
\]

\[
F_{2}^{(0)} = \sum_{i} e_i^2 \xi \left( q(\xi, Q^2) + \bar{q}(\xi, Q^2) \right)
\]

\[
\rho^2 = 1 + \frac{4M^2 x^2}{Q^2}
\]

Parton-hadron mismatch, ``collinear'' means up to pT=M, see also OPE

Georgi & Politzer, PRD 14 (1976); Barbieri et al., NP B117 (1976)
Definitions-convolutions

\[
\begin{align*}
    h_2(\xi, Q^2) &= \int_{\xi}^{1} du \frac{F_2^{(0)}(u, Q^2)}{u^2} \\
    h_3(\xi, Q^2) &= \int_{\xi}^{1} du \frac{F_3^{(0)}(u, Q^2)}{u} \\
    g_2(\xi, Q^2) &= \int_{\xi}^{1} du \ h_2(u, Q^2) = \int_{\xi}^{1} du \ \int_{u}^{1} dv \ \frac{F_2^{(0)}(v, Q^2)}{v^2} \\
    &= \int_{\xi}^{1} dv \ \int_{\xi}^{u_{\text{max}}=v} du \ \frac{F_2^{(0)}(v, Q^2)}{v^2} = \int_{\xi}^{1} dv \ (v - \eta) \frac{F_2^{(0)}(v, Q^2)}{v^2}
\end{align*}
\]
Corrections to structure functions at large Bjorken $x$

Some of the correction comes from the replacement:

$$ x \rightarrow \xi $$

Falling parton distribution functions, so small change in momentum fraction makes big change in structure function at large $x$.

NOTE: corrections to cross section not nearly factors of two.

Structure function TMC corrections

Comparison of full TMC to leading term, for different Q values.
These are ratios, not absolute sizes.

Structure function TMC corrections

Target Mass Correction, maximum of a couple of percent at 50 GeV, closer to 5% at 10 GeV.
Important kinematic regions are at low $Q$:

$$W^2 = Q^2 \left( \frac{1}{x} - 1 \right) + M^2$$
Low $Q$ important at low energy

MRST2004 PDFs with $Q_0^2 = 1$ GeV$^2$
with $Q$ frozen at $Q_0$ for low $Q$.

(Includes NLO QCD and TMC)
Help from electromagnetic scattering

Popular parameterization: Abramowicz, Levin, Levy and Maor (ALLM), here with the solid line. (hep-ph/9712415). This used 23 parameters.

Also shown: NLO+TMC and NNLO+TMC and SLAC data from Whitlow et al, Phys. Lett B (1990).

Fig. from Reno, PRD 75 (2006).
Help from electromagnetic scattering

\[ Q^2 = 4 \text{ GeV}^2 \]

\[ Q^2 = 0.5 \text{ GeV}^2 \]

\[ F_2(x, Q^2) \]

\[ E = 10, 5 \text{ GeV} \]

\[ W_{min}^2 = 2, 4 \text{ GeV} \]

Dashed: NLO + TMC with frozen Q, MRST2004
Couple of low $Q$ extrapolations of DIS

- Bodek-Yang-Park
- Capella, Kaidalov, Merino and Tranh Van: CKMT (see Reno, PRD75 (2006).)


Bodek, Yang, Park

- GRV 98 LO
- $x \rightarrow \xi_w$
- Multiply all PDFs by K factors

$$K_{sea}(Q^2) = \frac{Q^2}{Q^2 + C_s}$$

$$K_{valence}(Q^2) = \left[ 1 - G_D^2(Q^2) \right] \times \left( \frac{Q^2 + C_{v2}}{Q^2 + C_{v1}} \right)$$

- freeze low Q

$$F_2(x, Q^2 < 0.8) = K(Q^2) \times F_2(\xi, Q^2 = 8)$$

- fit effective PDFs

$$\xi_w = \frac{2x(Q^2 + M_f^2 + B)}{Q^2[1 + \sqrt{1 + (2Mx)^2/Q^2]} + 2Ax}.$$
Alternative parameterization of electromagnetic structure function: CKMT

7 parameters in

\[ F_2(x, Q^2) = F_2^{sea}(x, Q^2) + F_2^{val}(x, Q^2) \]

\[ = A x^{-\Delta(Q^2)} (1 - x)^{n(Q^2)+4} \left( \frac{Q^2}{Q^2 + a} \right)^{1+\Delta(Q^2)} \]

\[ + B x^{1-\alpha_R}(1 - x)^{n(Q^2)} \left( \frac{Q^2}{Q^2 + b} \right)^{\alpha_R} \]

\[ \times \left( 1 + f(1 - x) \right) \]
Valence component

CKMT fit $\alpha_R = 0.4250$ and $b = 0.6452$ GeV$^2$.

$$F_2^{val}(x, Q^2) = Bx^{1-\alpha_R}(1 - x)^n(Q^2) \left( \frac{Q^2}{Q^2 + b} \right)^{\alpha_R} \left( 1 + f(1 - x) \right)$$

$B = B_u$ is calculated to be 1.2064, $f = B_d/B_u = 0.15$ is also calculated. They are calculated invoking valence counting rules at $Q^2 = 2$ GeV$^2$. Also fit is $c = 3.5489$ GeV$^2$ in

$$n(Q^2) = \frac{3}{2} \left( 1 + \frac{Q^2}{Q^2 + c} \right)$$
Sea component

CKMT fit $A = 0.1502$ and $a = 0.2631$ GeV$^2$.

$$F_2^{sea}(x, Q^2) = Ax^{-\Delta(Q^2)}(1 - x)^{n(Q^2)+4} \left(\frac{Q^2}{Q^2 + a}\right)^{1+\Delta(Q^2)}$$

Also fit is $\Delta_0 = 0.07684$ and $d = 1.1170$ GeV$^2$ in

$$\Delta(Q^2) = \Delta_0 \left(1 + \frac{2Q^2}{Q^2 + d}\right)$$

$\Delta_0$ is similar to power law in generalized vector meson dominance at low $Q^2$, where it is pomeron dominated.
Convert to neutrino scattering

\[ F_2^{ep} = \frac{4}{9} x (u + \bar{u}) + \frac{1}{9} x (d + \bar{d} + s + \bar{s}) \]

\[ F_2^{\nu p} = 2x(d + s + \bar{u}) \]

Different weight to the sea and valence distributions when considering neutrino scattering rather than EM scattering.
CKMT for neutrinos

- Expect that the underlying non-perturbative process is governed by the same power law and form factor:
  \[ \Delta(Q^2) \approx 0.08, \quad (Q^2/(Q^2 + a))^{1+\Delta} \]
  Match A at small x where sea dominates.

- For the valence part, recalculate B and f:
  \[ B_\nu = 2.695, \quad f_\nu = 0.595 \text{ at } Q^2 = 2 \text{ GeV}^2 \]

- Valence x and Q dependence shouldn’t change between electromagnetic and charged current scattering.

- For \( F_1 \), use a parameterization of R from Whitlow et al, Phys. Lett. 1990
CKMT for neutrinos

- For F3, use

\[
F_3(x, Q^2) = \left[ \frac{A_\nu}{15} x^{-\Delta(Q^2)} (1 - x)^{n(Q^2)+4} \left( \frac{Q^2}{Q^2 + a} \right)^{1+\Delta(Q^2)} \right]
\]

\[+ \quad B_\nu x^{1-\alpha_R} (1 - x)^{n(Q^2)} \left( \frac{Q^2}{Q^2 + b} \right)^{\alpha_R} \]

\[\times \quad (1 + f_\nu(1 - x)) \right] / (1.1x) \]

- The denominator of 1.1 adjusts the integral of the valence quark part to give a Gross-Llewellyn-Smith sum rule results of 3x0.9 (QCD corrected.)

\[
F_3^{\nu N} = \frac{1}{2} \left[ F_3^{\nu p} + F_3^{\nu n} \right]
\]

\[
= \frac{1}{2} \left[ 2(d + s - \bar{u}) + 2(u + s - \bar{d}) \right]
\]

\[
= d - \bar{d} + u - \bar{u} + 2s = d_v + u_v + 2s
\]

\[
\int_0^1 dx \ F_3^{\nu N} \simeq \int dx (d_v + u_v) = 3
\]
CKMT and BYP

Solid lines: BYP, dashed: CKMT
Muon neutrino charged current cross section

\[ W_{\text{min}}^2 = 2 \text{ GeV}^2 \]

\[ W_{\text{min}}^2 = 4 \text{ GeV}^2 \]

Low Q extrapolations, from NLO+TMC, with BYP and CKMT

Reno, PRD75 (2006)

High energy K-factors, see, e.g., Jeong & Reno, PRD81 (2010)
Muon anti-neutrino charged current cross section

\[ \sigma_{\nu N} = \begin{cases} \text{Low Q extrapolations,} \\ \text{from NLO+TMC, with} \\ \text{BYP and CKMT} \end{cases} \]

\[ W_{\min}^2 = 2 \text{ GeV}^2 \]

\[ W_{\min}^2 = 4 \text{ GeV}^2 \]

Reno, PRD75 (2006)
DIS only-CKMT extrapolation

Gluck, Jimenez-Delgado, Reya PDFs, NLO + TMC, CKMT patched below critical $Q$

$Q_c^2 = 2 \text{ GeV}^2$

Recent update, Y.S. Jeong and M.H. Reno (in preparation)
Few pion is approximately comparable to QE
# Tau neutrino cross sections

<table>
<thead>
<tr>
<th>Energy [GeV]</th>
<th>$\sigma_{\nu N}$ [10$^{-38}$ cm$^2$]</th>
<th>$\sigma_{\bar{\nu} N}$ [10$^{-38}$ cm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.916 (1.26, 0.690)</td>
<td>0.291 (0.574, 0.160)</td>
</tr>
<tr>
<td>$10^{1.25}$</td>
<td>3.77 (4.22, 3.44)</td>
<td>1.48 (1.90, 1.21)</td>
</tr>
<tr>
<td>$10^{1.5}$</td>
<td>10.4 (10.9, 9.97)</td>
<td>4.43 (4.93, 4.05)</td>
</tr>
<tr>
<td>$10^{1.75}$</td>
<td>23.7 (24.3, 23.2)</td>
<td>10.6 (11.2, 10.2)</td>
</tr>
<tr>
<td>$10^2$</td>
<td>48.9 (49.5, 48.4)</td>
<td>22.8 (23.4, 22.3)</td>
</tr>
<tr>
<td>$10^3$</td>
<td>$5.69 \times 10^2$</td>
<td>$3.02 \times 10^2$</td>
</tr>
<tr>
<td>$10^4$</td>
<td>$4.30 \times 10^3$</td>
<td>$2.76 \times 10^3$</td>
</tr>
</tbody>
</table>

$W_{\text{min}} = 1.4$ GeV ($M$, 1.7 GeV)
y-distributions

DIS only, with $W_{\text{min}}=1.7 \text{ GeV}$

Jeong and Reno, in preparation.
y-distributions

DIS only, with $W_{\text{min}} = 1.7$ GeV
Conclusions – have focused mainly on DIS

- 5-15 GeV range is most uncertain theoretically because of the mix of inclusive and exclusive processes.
- The low-Q region in DIS important for low energies – the new PDFs with lower Q0 help.
- TMC important at the level of about 5% at 10 GeV neutrino energies.
- Generally, antineutrino scattering has larger uncertainties than neutrino scattering because of lower Q values.
Low $Q$ extrapolation

- Electromagnetic

\[ \sigma(\gamma^*p) = \frac{4\pi^2\alpha}{Q^2} F_2(x, Q^2) \Rightarrow F_2 \to Q^2 \to 0 \text{ const} \cdot Q^2 \text{ or } F_2 \sim C \frac{Q^2}{Q^2 + A^2} \]

- Weak interactions

\[ F_2 \sim C' \left( \frac{Q^2}{Q^2 + A^2} + \frac{Q^2 + D^2}{Q^2 + B^2} \right) \]

CCFR/NuTeV found $F_2(x, Q^2 = 0) = 0.21 \pm 0.02$

B. Fleming et al, PRL 86 (2001)
Recent measurements—zoom in

Black error bar statistical, shaded systematic. For 10 GeV, this is about +10%.

There is some energy dependence to the cross section…

MINOS, Adamson et al, PRD81, 072002 (2010)
Ratio of neutrino/antineutrino CC

MINOS, Adamson et al, PRD81, 072002 (2010)