Towards Scale-invariant Cyclic universes

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arXiv:0801.1315 [hep-th]
Standard Model of Cosmology

\[ ds^2 = a^2(\tau)[-(1 + \Phi(\tau, x))d\tau^2 + (1 - \Phi(\tau, x))dx^2] \]

**Key ingredients**

- GR + Homogeneous Isotropic cosmology + Perturbations
- Inflation \(\rightarrow\) Radiation \(\rightarrow\) Matter (DM + baryons) \(\rightarrow\) Dark Energy

Concordant \(\Lambda\)CDM model
Why do we still have our jobs?
Finally we know for sure that we almost know nothing
Inflaton, Baryons, Dark Matter, Dark Energy????

Two Attitudes

- SM is fine, try to address the hard questions
- Search for alternative Non-singular (QG) cosmology
  - Cosmology is an ultimate magnifier
    1. Length
      \[ H_0^{-1} \xrightarrow{\text{radiation}} \frac{T_0}{T_{GUT}} H_0^{-1} \xrightarrow{\text{inflation}} e^{-N} \left( \frac{M_p^2}{T_{GUT} T_0} \right) l_p = e^{-(N-75)} l_p \]
    2. Energy density can become Plackian
    3. Spatial curvature can become Planckian

- Observable Quantum Gravity Effects may be testable
  already (WMAP) and/or near future (Planck)
  1. Spectrum
  2. Gravity waves
  3. Non gaussianity and non adiabaticity
Non-singular Cosmologies

- “Effective” FLRW cosmologies a good description: LQC + BKL
- Look into the “eternal past”

\[ R \sim H^2 \sim \left( \frac{\dot{a}}{a} \right)^2 \]

Bouncing Universe
Novello, Salim; Melnikov, Orlov, 70’s
Gaperini, Maggiore & Veneziano
(pre-big bang) ’97

Emergent Universe
Ellis & Maartens ’04

Cyclic Universe
Einstein, Freedman, Tolman, Lemaître, 30’s
Bondi, Gold, Narlekar, Hoyle (steady state) 50’s
Steinhardt & Turok (ekpyrotic), ‘02
Freese et al.; Frampton & Baum (phantom)
Plan of the talk

- Virtues and Challenges of cyclic cosmology
- Tracking perturbations around Bounce
- New “emergent cyclic Universe”
- Cyclic Inflation
Challenges & Virtues of Cyclic Universes

I. Nonsingular geodesically complete Universe

- Consistent (ghostfree) Bounce
  - LQC
  - p-adic/SFT inspired Non-local modifications of gravity [Seigel, Mazumdar, YT]
  - BCS Gap energy: [Alexander, & YT]
- Coupling of fermions to gravity $\Rightarrow$ four fermion interaction
- Attractive $\Rightarrow$ negative energy required for bounce
- Gap energy $\rightarrow$ chemical potential $\rightarrow$ number density $\rightarrow$ volume
- Nontrivial volume dependence can temporarily violate DEC
II. **Turn-around**
- Spatial curvature (no Dark Energy)
- Scalars, phantom

III. **Classic puzzles:**
- Horizon: All
- Flatness, Largeness, Entropy: Emergent cyclic models
- Homogeneity/Isotropy: Some aspects we will be able to address

IV. **Black hole over-production & Dark energy**
- Quintessence (ekpyrotic) makes BH’s dilute
- Phantom makes BH’s disintegrate

V. **Generating Scale-invariant Perturbations**
- Scalar field fluctuations [Steinhardt et.al]
- Stringy thermal fluctuations [Brandenberger et. al]
- “New” ideas

VI. **Tolman’s Entropy or the “Super Big Bang” Problem**
- Thermal UV phase (Hagedorn phase) exists +
- BB singularity problem is solved
Transferring fluctuations via bounce

**Inflation Basics**  \( V''(\Phi) \approx 0, \ a \sim -1/H_\tau \)

\[
\sigma_k'' + \left( k^2 - \frac{a''}{a} \right) \sigma_k = 0 \quad \sigma \equiv \frac{\phi}{a}
\]

- **Sub-Hubble**
  \( H \ll k \quad \Rightarrow \quad \sigma_k \sim k^{-1/2} e^{i k \tau} \)

- **Super-Hubble**
  \( H \gg k / a \quad \sigma_k \sim A(k) a(\tau) \)

- **Matching at Hubble crossing**
  \( H = k / a \quad \Leftrightarrow \quad k \sim 1 / |\tau| \quad \varphi_k \sim k^{-3/2} \)

- **Power Spectrum**
  \( P_\phi \sim k^3 \left| \varphi_k \right|^2 \sim \text{const.} \)

**Ekpyrotic**

- Generates scale-invariant spectrum in the “growing mode” during contraction phase.
Perturbations in Nonsingular Bounce

- Interested in constant mode in expanding branch
- Mode matching ambiguities
- Physics at bounce not known/singular
- Numerical solutions tricky

How to avoid the pit-falls?

- GR equation valid away from bounce

\[ \Phi''_k + 3(1 + \omega)H\Phi'_k + [\omega k^2 + 2H' + (1 + 3\omega)H^2] \Phi_k = 0 \]

Solutions known: \((-\infty, \tau_p)\) \& \((\tau_p, \infty)\)

- In super-Hubble spatial derivatives not important
  - If no inflation \(k_{ph} \ll \tau_p^{-1} \sim O(M_p)\)
  - Near bounce also like super-Hubble, even if \(H \to 0\)
  - Higher order spatial derivatives can also be neglected
  - Perturbation eqn. only depends on time,
  - find fluctuations for scale-factor \(\Leftrightarrow\) only need homogeneous isotropic cosmology
- Matching in overlapping regions \((-\tau_k, -\tau_p)\) \& \((\tau_p, \tau_k)\)
An example

Assumption
- Perturbation equation not effected by new physics
- Non-local physics changes global evolution
- Does not effect perturbations
- Casimir Energy, Gap energy

Results
- Depends on the new physics (background)
- Generally modes mix! Good news for ekpyrotic scenarios
- Modes can even switch
  - Modes switch at $\omega \approx 5.314$,
  - Mixes when $\omega >> 1$ (ekprotic limit)
Emergent Cyclic Universe

Tolman’s Entropy Problem

- Entropy is monotonically increasing
  - Universe at most quasi-periodic
  - Entropy (Energy, period) vanishes in a finite time in the past
  - Beginning of time – back to square 1

- Thermal Hagedorn Phase, \( T = T_H \approx M_S \)
  - All string states in thermal equilibrium entropy constant
  - Below critical temperature, massive states decouple, entropy produced.
  - As cycles shrink, universe is hotter less time in entropy producing
    more time in
    \[
    \Delta S \rightarrow 0 \quad \Rightarrow S_n \rightarrow S_{-\infty} \neq 0
    \]
  - Cycles asymptote to a periodic evolution

non-Hagedornic phase

Hagedorn phase
Stringy Toy Model

- **Hagedorn half**
  \[ H^2 = \frac{1}{3} \left[ \rho_{\text{hag}} - \rho_{\text{cas}} + \rho_{\text{curv}} \right] = \frac{1}{3} \left[ \frac{S}{a^3} - \frac{\Omega_c}{a^4} - \frac{\Omega_k}{a^2} \right] \]
  - Eternally periodic universe
    \[ a(\tau) = \frac{S - \sqrt{S^2 - 4\Omega_c \Omega_k \cos \nu \tau}}{2\Omega_k} \quad \nu = \frac{T_H^2}{M_p \sqrt{\frac{\Omega_k}{3}}} \]

- **Non-Hagedorn half (no energy exchange)**
  - Hagedorn matter = massless (r)+massive (m)
  - \( S \rightarrow \Omega_m \quad \& \quad \Omega_c \rightarrow \Omega_c - \Omega_r \quad \Rightarrow \quad a_{\text{max}} \sim \sqrt{\Omega_r} \)

- **Gluing the two halves** (\( \Omega_r, \quad \Omega_m, \quad a_{\text{tran}} \))
  - Entropy is conserved
  - Matter and radiation was in equilibrium till the transition

- Phenomenological Input:
  \[ \mu \equiv \frac{\rho_m}{\rho_r} \sim 10^{-22} \ll 1 \]
  \[ \Omega_r \sim S^{4/3} \Rightarrow a_{\text{max}} \sim S^{2/3} \quad a_{\text{tran}} \sim S^{1/3} \]
Energy Exchange & Entropy Production

- Matter converted to radiation [Tolman, Barrow et al.]
  \[ \dot{\rho}_r + 4H\rho_r = T_H^4 s \]
  \[ \dot{\rho}_m + 4H\rho_m = -T_H^4 s \]
  \[ \dot{S}_{tot} = a^3 S \left( \frac{T_H}{T_r} - \frac{T_H}{T_m} \right) \]

- Consistent with 1st & 2nd law of thermodynamics
- Conserves total
- Breaks time-reversal symmetry
- Exchange function, \( s(a, \Omega) \)
- Small cycle limit, \( s \sim \text{const.} \)

Crucial Difference

- Singular bounce \( \Gamma \) interaction cannot keep up with \( H \rightarrow \infty \)
- Thermal equil cannot be maintained entropy production during bounce
- Nonsingular bounce \( H \) is finite, a thermal Hagedorn phase can exist.
Cyclic Inflation

Can we mimick inflation?

- Entropy production by decay of massive particles:
  \[
  \left( \frac{S_{n+1}}{S_n} \right) = \left( \frac{S_r}{S_m} \right) \sim \frac{\rho_r^{3/4} V}{\rho_m VM^{-1}} \sim \frac{T_H}{T_d} \equiv \kappa
  \]
  - Entropy increases by a constant factor
  - If decay time > Period, $\kappa$ smaller
  - So does the scale factor!
  - Energy density remains same
  - Inflation over many cycles

- Massless scalar field will see inflation on an average, if
  \[\tau H_{av} \ll 1\]
More detail requirements

- With curvature turnaround $\tau$ increases
- -ve consmological constant $\sim -\lambda^4 \Rightarrow \tau \sim \frac{T_H}{\lambda^2}$ \quad $\lambda < T_H$

$$H_{av} = \frac{\int H \, dt}{\int dt} = \frac{\ln \left( \frac{a_{n+1}}{a_n} \right)}{\tau} \Rightarrow \ln \left( \frac{a_{n+1}}{a_n} \right) = \frac{\ln \kappa}{3} << 1$$

- Over many many many cycles we realize inflation

How to exit?

- Introduce a potential
  - Kinetic energy at end of contraction depends on slope:
    - Total energy can go from $-ve$ to $+ve$
    - Can zoom past the minimum and into $+ve$ potential region
    - Can even mimick quintessence, not necessary.
Thermal fluctuations

**Spectrum** [Peebles’93, Pogosian & Magueijo]

- **Mechanism**
  - Energy fluctuation in a given volume related to Power spectrum
  - White noise spectrum

\[
\delta_L^2 = \left. \frac{\Delta E^2}{E^2} \right|_L = \left( \frac{d^2 \ln Z}{d \beta^2} \right) \left/ \left( \frac{d \ln Z}{d \beta} \right)^2 \right. = \frac{T^2}{\rho^2 L^3} \frac{\partial \rho}{\partial T} \sim \frac{1}{(TL)^3}
\]

- Thermal correlations only for sub-Hubble modes
- Hubble Crossing
  - Conformal radiation leads to scale invariance
- \( P(k) \) transferred to gravity, \( P_\Phi(k) \)
  - Super Hubble governed by \( \Phi_k \)
- **Robust and general mechanism (no fine-tuning):**
  - At Hubble crossing validity of GR
  - Radiation domination
  - Contraction so that modes leave the Hubble radius
  - No mode mixing, \( \Phi_k \) remains constant
Amplitude

- The problem for symmetric bounce
- Consider scale which enters Hubble radius at $T_{eq}(1\text{Mpc})$

$$\delta_L^2 = \frac{1}{g^*_e (LT_L)^3} \sim \frac{1}{g^*_e} \left( \frac{H_L}{T_L} \right)^3 \sim \sqrt{g^*_e} \left( \frac{T_L}{M_p} \right)^3 \ll 10^{-8} \Rightarrow T_L \sim 10^{13} \text{Gev}$$

- Exactly the right scenario to “amplify” perturbations

$$k = T_L^2 a_L = T_{eq}^2 a_{eq} \quad \& \quad S \sim (T a)^3 \sim T^{-3}$$

$$\left( \frac{S_0}{S_{-1}} \right) \equiv \kappa = 10^{67}$$

- $a_p$ is increasing $\lambda_{ph}$ is decreasing with cycles
  $\Rightarrow$ universe has to expand more for the same mode to exit
  we need
Thermal Equilibrium in Hagedorn phase [Frey et.al, Hindmarsh & Skliros]

- **Small cycles:** Thermal equilibrium to be maintained
  \[ H < \Gamma_{int} \]
  \[ \Gamma_{int} \sim T_H g_s^2 n \]
  \[ T_H \sim 10^{-3} M_p \]
  \[ g_s^2 > 10^{-5} \]

- **Large cycles:** \[ H \propto S^2 \]
  Thermal equilibrium around bounce short lived
  Near bounce large amounts of entropy produced

Entropy Production around Bounce

- **Naïve Estimate:** when radiation converts to Hagedorn matter
  \[ \left( \frac{S_0}{S_{-1}} \right) = \left( \frac{S_H}{S_r} \right) \sim \frac{\rho_r V T_H^{-1}}{\rho_b^{3/4} V} \sim \rho_b^{1/4} \sim S_{-1} \]

- **Holographic Saturation:** Can't trust physics at super-Planckian energy densities
  Treat bounce at black box, use holographic entropy bound
  If saturated we get the same estimate!

  \[ S_0 \sim 10^{134} \quad \& \quad S_{-1} \sim 10^{67} \]

  \[ S \leq area \sim \left( \frac{E}{M_p} \right)^2 \Rightarrow S_0 \leq S_{-1} \left( \frac{T_H}{M_p} \right)^2 \]
Dark Energy and Black Hole Problem

- Add $\Lambda \sim \text{(mev)}$ there exists a last (our) cycle! DE phase

- Previous cycles very short
  - no matter domination, no LSS
  - no junk (BH/inhomogeneities) from previous cycle
  - thermal density fluctuation very small

- Toy Model can be extended to ekpyrotic/phantom type late cycles
Summary

The story so far?
- Began as string size, curved universe, in almost periodic & almost Hagedornic phase: emerging phase
- (a) Constant cycle “inflationary” phase -> graceful exit to long lived cycle
- (b) With entropy production, Hagedorn phase becomes shorter,
  - large entropy production starts
  - Universe highly asymmetric and large
  - Very large cycles (like ours), curvature gives way to Dark Energy

Problems addressed
- Singularities: BBS & super BBP
- Classic Problems: horizon, flatness, entropy, largeness
- Late time: inhomogeneity/BH overproduction & DE
- Perturbations: spectrum and amplitude

Needs investigation
- Initial homogeneity/isotropy, Mix master behaviour
- Hagedorn physics: Non-equilibrium dynamics, Casimir Energy, Decay rate...
- Mode mixing during Bounce? Is $\Phi$ really constant during bounce?

Predictions/Tests
- Gravity Waves, Non-gaussianity...
Precision Cosmology

Cosmic Microwave Background
We need an asymmetric bounce, the comoving scale has to exit the Hubble radius much earlier.

This is precisely what we have via entropy production.
Quantum Gravity: A toy Model

Motivation

- **Stringy**
- Dual Field theory action for strings on Random Lattice
  \[ \hat{S} = \int d^D x \ tr \left[ \frac{1}{2} \phi e^{-\alpha' \Box / 2} \phi + G^{m-2} \phi^n \right] \]
  Linear Regge trajectories: Confinement [Grisaru, Siegel, Y.T.]
- Tachyons in open SFT and p-adic string theory has similar form
- **Higher Derivative but Ghost free**
  \[ S = \int d^4 x \ \phi \Box (\Box + m^2) \phi \Rightarrow \Box (\Box + m^2) \phi = 0 \]
  \[ \Delta(p^2) = \frac{1}{p^2(p^2+m^2)} \sim \frac{1}{p^2} - \frac{1}{(p^2+m^2)} \]
- **Non-singular UV & IR behavior**
  Weinberg’s “Asymptotic safety”
- **Non-perturbative Quantum gravity**
  Close to Planck scale, all terms important
Model

- **Action**
  \[ S = \frac{M_p^2}{2} \int d^4 x \sqrt{-g} \left[ R + \sum_{n=0}^\infty c_n R^{\Box^n} R \right] \quad c_n \sim \frac{1}{M^{2n+2}} \]

- **Generalized \``Einstein’s\'' Field Equations**
  \[ \tilde{G}_{\mu\nu} \equiv G_{\mu\nu} + \sum_{n=0}^\infty G^n_{\mu\nu} = T_{\mu\nu} \]
  \[ G^n_{\mu\nu} \sim c_n (\Box^{n+1} R + \Box^p R \Box^m R) \]

- **Conservation Equation**
  \[ \nabla^\mu \tilde{G}_{\mu\nu} = 0 \]

  For Cosmology \[ \tilde{G}_{00} = 0 \] equation suffices

- **Exact Bouncing solutions!** \[ a(t) = \cosh(\lambda t) \]

  \[ \Lambda \neq 0, \quad \rho_{rad} \sim M_s^2 M_p^2 \quad \& \quad \lambda \sim M_s \]
- **Can Inflation be past-eternal?**

  - Open or flat: geodesically incomplete
  - Closed: geodesically complete

- **Big Bounce:** $H = 0 \ & \ \ddot{a} > 0$

  $$H^2 = \frac{\sum \rho_I}{3M_p^2}$$

- Flat/Open: DEC, WEC violation
Plan for the rest of the Talk

- Consistent (ghostfree) modification of Gravity
  LQC, Bouncing and cyclic Universes
  Non-perturbative gravity $\Rightarrow$ Bouncing Universe
- Consistent (ghostfree) modification of Matter sector
  Ghost Condensation [Arkani-Hamed et.al.,Khoury et.al.]
  Casimir Energy and an “emergent cyclic Universe”
  Hagedorn Physics & Tolman’s Entropy Problem
- Gap Energy and cosmological BCS Condensation [Alexander & Vaid]

- Generating Scale-invariant Perturbations
  Scalar field fluctuations [Steinhardt et.al]
  Stringy thermodynamic fluctuations [Brndenberger,Nayeri & Vafa]
Thermodynamic Fluctuations during Hagedorn Phase

Energy to Power Spectrum

- Energy to density fluctuations:
  \[ |\delta\rho_k|^2 \sim k^3 |\delta E(r \sim k^{-1})|^2 \]

- Energy fluctuations from Heat Capacity
  \[ Z \sim \sum e^{-\beta E} \Rightarrow \delta E(r) \sim T^2 C_v \sim \frac{T}{T_H - T} r^2 \]

- Matter fluctuations to metric perturbations
  \[ \nabla^2 \Phi = 4\pi G \delta \rho \Rightarrow |\Phi_k|^2 \sim k^{-4} |\delta \rho_k|^2 \]

- Power Spectrum
  \[ P_\Phi \sim k^3 |\Phi_k|^2 \sim \frac{T}{T_H - T} \]
CMB Spectrum: Minimal Requirements

- Fluctuations should come from massive modes and not massless (radiation)

\[ C_{\text{rad}} \ll C_{\text{massive}} \quad \Rightarrow \quad \frac{\Delta T}{T_H} < 10^{-30} \]

- Amplitude:

\[ \delta_{CMB}^2 \sim 10^{-10} \sim \left( \frac{M_s}{M_p} \right)^4 \frac{T_H}{\Delta T} \]

\[ \frac{T_H}{\Delta T} = 10^{30} \quad \Rightarrow \quad \frac{M_s}{M_p} \sim 10^{-10} \]

- Spectral tilt:

\[ |\eta_s - 1| \approx 10^{-60} \left( \frac{M_s}{\lambda} \right)^2 \frac{T_H}{\Delta T} \]

Typically, one obtains almost perfect scale-invariance!
- **t’ Hooft dual to string theory**
- **Polyakov action:**
  \[ S = \int \frac{d^2 \sigma}{2\pi} \sqrt{-h} \left[ \frac{h^{\alpha\beta}}{2\alpha'} (\partial_\alpha X)(\partial_\beta X) \right] \]

- **Strings on Random lattice** [Douglas, Shenker]
  \[ S = \sum_{ij} (X_i - X_j)^2 \]
  \[ \Rightarrow Z = \int \mathcal{D}h \mathcal{D}X \ e^{-S} = \sum \int d^D X \ \prod_{ij} e^{-\frac{1}{2\alpha'}(X_i-X_j)^2} \]

- **Dual Field theory action**
  \[ \hat{S} = \int d^D x \ \text{tr} \left[ \frac{1}{2} \phi e^{-\alpha' \Box/2} \phi + G^{m-2} \phi^n \right] \]

**Linear Regge trajectories: Confinement** [Grisaru, Siegel, Y.T.]
Finite Order Gravity

Improved UV behaviour: 4th Order Gravity

\[ S = \int d^4x \sqrt{-g}(R + c_0 R^2 + b_0 C^2) \]

even Renormalizable \cite{Stelle, 1978}

Asymptotically free + Renormalizable!

Unfortunately \( b_0 \neq 0 \Rightarrow \text{Ghosts} \)

If \( b_0 = 0 \) Asymptotic freedom, Renormalizability lost

- (Ghost + asymptotically) free gravity => NP gravity
Propagator

- **Scalar-Tensor Picture:** HD terms in $\phi$

\[
S = \int d^4x \sqrt{-g} \left[ e^{-\phi} R + \psi \sum_{i=0}^{\infty} c_i \Box^i \psi - \{\psi (e^{-\phi} - 1)\} \right]
\]

p-adic scalars in a curved background + dilaton?

- **Field Equation**

\[
(1 - 6 \sum_{i=0}^{\infty} c_i \Box^{i+1}) \phi \equiv \Gamma(\Box)\phi = 0 \Rightarrow \Delta(p^2) = \frac{1}{\Gamma(-p^2)}
\]

- **Ghost free if $\Gamma(\Box)$ has:**
  - a single zero, $R^2$ gravity
  - no zeroes, Gaussian’s

  \[
  \Delta(p^2) = \frac{1}{(p^2+m^2)}
  \]

  \[
  \Delta(p^2) = e^{-p^2/m^2}
  \]

- **Improved UV behaviour:**

  \[
  h \sim \frac{\text{erf}(r)}{r}
  \]
Transition to FRW, $\Lambda = 0$

**Late times**
\[ a(t) \to e^{\lambda t} \quad \& \quad \text{HD terms} \to \text{sech}^2(\lambda t) \sim e^{-2\lambda t} \to 0 \]
\[ \Rightarrow \text{Einstein Gravity & dS Universe} \Rightarrow \Lambda \neq 0 \]

**Near Bounce**
\[ G_{00} \to 0 \quad \text{but HD terms finite} \]

**Approximate Bounce**
- Small times: HD terms = radiation
  
  We found ghost free examples
- Transition: HD terms $\sim G_{00}$
- Large times: FRW cosmology, HD terms $<< 0$

\[ a(t) \sim t^{1/2}, \quad G_{00} \sim \frac{1}{t^2}, \quad \tilde{G}_{00}^n \sim \frac{1}{t^{2(n+1)}} \]
**Asymptotic Safety** [Weinberg, 1976]

- Renormalizability replaced by asymptotic safety
- Quantum behavior captured by RG flow

\[ \mu \frac{dg(\mu)}{d\mu} = ag^2(\mu) \quad (a > 0) \quad g(\kappa\mu) = \frac{g(\mu)}{1 - ag(\mu)\ln\kappa} \]

- Asymptotic safety = non-singularity → UV fixed point

4d gravity ⇒ \( G_N \) → 0 asymptotic freedom

- Although ghost free finite HD gravity theories exist, (Ghost + asymptotically) free gravity ⇒ NP gravity

- Quantum Gravity actions (closed under renormalization flows) contains specific infinite series of HD terms: [Krasnov, gr-qc/0703002]

  Equivalent to “second order theory”, no IVP
Hagedorn Phase

Qualitative Behaviour

- Close to $T = T_H \sim M_S$ massive (winding) string states are excited. Pumping energy doesn’t increase temperature, produces new states.

- Thermodynamics no longer determined by massless modes:
  $$E = T_H S - bV T_H^{d+1} + ...$$

- Cosmological Evolution: Entropy is constant
  Energy, Temperature remains approximately constant
  $$\frac{\Delta T}{T_H} \equiv \frac{T_H - T}{T_H} \sim \exp\left[-\frac{E}{V^{2/3} T_H^3}\right]$$

- Transition to radiation occur when $S \sim V T_H^d$
Cosmological BCS Condensation

- BCS Theory
- Free theory, fermions filled upto Fermi-sea
- Attractive four-fermion coupling contributes to negative energy.
- Vacuum gets a negative shift with the formation of mass-gap.
- Auxillary field
- Integrate the fermions, use mean field theory to get non-perturbative potential for Delta
- Gap Equation
**Potential**

- **Trace equation (Lorentz gauge)** \[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \]

\[ \Rightarrow \quad \widetilde{G} = -\frac{1}{2} \Box (1 - 6 \sum_{i=0}^{\infty} c_{i} \Box^{i+1}) h = -\frac{1}{2} \Box \Gamma(\Box) h \]

- **Potential for h**

\[ \widetilde{G} \sim -m \delta(r) \Rightarrow h(r) \sim \frac{1}{r} \int_{-\infty}^{\infty} dp \frac{p}{p^{2} \Gamma(-p^{2})} e^{ipr} \equiv \frac{G_{N}(r)}{r} \]

- **AF:** falls off exponentially \[ \Rightarrow \quad G_{N}(r) \to r \]

- **Newtonian Limit** \[ \Gamma(-p^{2}) \xrightarrow{p \to 0} 1 \quad \Rightarrow \quad G_{N}(r) \to \text{const.} \]

- **Example:** \[ \Gamma(\Box) = e^{-\Box} \Rightarrow h(r) \sim \frac{\text{erf}(r)}{r} \]
Big Bang Singularity

- In GR at $t = 0$ we encounter a singularity

$$R, \Box R, \rho \to \infty$$

Closed Geometry | Open Geometry | Flat Geometry

- SEC
  - $\rho + 3p < 0$

- SEC, DEC, WEC
  - $\rho < 0$
Non-singular Bounce

- **Ansatz:** Find $a(t)$ such that $\Box R \sim R$
  
  \[ (...R(t) + (...)R^2(t) \sim \text{matter sources} \]

  Reduces to solving algebraic equation

- **Hyperbolic Bounce** $a(t) = \cosh(\lambda t)$ works!

- **Evolution**

  \[
  \tilde{G}_{00} = T_{00} = \frac{1}{3} (\Lambda + \rho_{\text{rad}})
  \]

- **Solutions exist for asymptotically and ghost free theories:**

  $\omega = \frac{1}{3} \Rightarrow \rho_{\text{rad}} \sim \frac{1}{a^4}$ \& \( a \sim t^{1/2} \)