CFTs with O(N) and Sp(N) Symmetry and Higher Spins in (A)dS Space

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Based mainly on

- L. Fei, S. Giombi, IK, arXiv:1404.1094
- L. Fei, S. Giombi, IK, G. Tarnopolsky, arXiv:1411.1099
- S. Giombi et al, arXiv:1306.5242
From D-Branes to AdS/CFT

• Stacking D-branes and comparing the gauge theory living on them with the curved background they create led to the gauge/gravity duality.

• In case of N D3-branes, have SU(N) gauge theory with $\mathcal{N}=4$ SUSY.

$$ds^2 = \left(1 + \frac{L^4}{r^4}\right)^{-1/2} \left(- (dx^0)^2 + (dx^i)^2\right) + \left(1 + \frac{L^4}{r^4}\right)^{1/2} (dr^2 + r^2 d\Omega_5^2)$$

• Absorption of closed strings, near-extremal entropy, the “3/4 problem” IK, Gubser, Peet, Tseytlin, …

$$s = \frac{\pi^2}{2} N^2 T^3$$

• Zoom in on the throat: $AdS_5 \times S^5$ with $L^4 = g_{YM}^2 N \alpha'^2$

Maldacena
From Strong to Weak

• Theory local on AdS radius scale $L$, since the string scale $<< L$ when $\lambda$ is large.

• Some non-BPS tests using integrability and SUSY localization to get at the strongly coupled gauge theory.

• As $\lambda->0$ the dual geometry becomes strongly curved, but another symmetry emerges: the higher spin symmetry. Sundborg

• Bilinear currents in SYM become conserved; dual to higher spin gauge fields in AdS.

• Theory should become non-local on AdS radius scale.
Try to Simplify

• In CFTs with adjoint fields, lots of additional operators.
• There are simpler CFTs where dynamical fields are in the fundamental of O(N) or U(N) and the only operators are bilinears. IK, Polyakov
• Wilson-Fisher O(N) critical points in d=3:
\[ S = \int d^d x \left( \frac{1}{2} (\partial \phi^i)^2 + \frac{\lambda}{4} (\phi^i \phi^i)^2 \right) \]
• Even simpler: the O(N) singlet sector of the free theory.
• Conserved currents of even spin
\[ J_{(\mu_1 \ldots \mu_s)} = \phi^a \partial_{(\mu_1} \ldots \partial_{\mu_s)} \phi^a + \ldots \]
All Spins All the Time

• Similarly, can consider the U(N) singlet sector in the d-dimensional free theory of N complex scalars. There are conserved currents of all integer spin.

• The dual AdS\textsubscript{d+1} description must consist of massless gauge fields of all integer spin, coupled together.

\[
\text{Spectrum : } \begin{align*}
    s = 1, 2, 3, \ldots, \infty & \quad \text{gauge fields} \\
    s = 0, \quad m^2 = -2(d - 2) & \quad \text{scalar}
\end{align*}
\]

• Vasiliev and others have constructed the classical EOM for some such interacting theories.
Interacting CFT’s

• A scalar operator $\mathcal{O}(x^\mu)$ in $d$-dimensional CFT is dual to a field $\Phi(z, x^\mu)$ in AdS$_{d+1}$ which behaves near the boundary as $z^\Delta$

• There are two choices

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\left(\frac{d}{2}\right)^2 + m^2}$$

• If we insist on unitarity, then $\Delta_-$ is allowed only in the Breitenlohner-Freedman range

$$-(d/2)^2 < m^2 < -(d/2)^2 + 1$$

IK, Witten
• Flow from a large N CFT where $O(x^\mu)$ has dimension $\Delta_-$ to another CFT with dimension $\Delta_+$ by adding a double-trace operator. Witten; Gubser, IK

• Can flow from the free d=3 scalar model in the UV to the Wilson-Fisher interacting one in the IR. The dimension of scalar bilinear changes from 1 to 2 +O(1/N). The dual of the interacting theory is the Vasiliev theory with $\Delta=2$ boundary conditions on the bulk scalar.

• The 1/N expansion is generated using the Hubbard-Stratonovich auxiliary field.

\[
S = \int d^d x \left( \frac{1}{2} (\partial \phi^i)^2 + \frac{1}{2} \sigma \phi^i \phi^i - \frac{\sigma^2}{4\lambda} \right)
\]
• In $2<d<4$ the quadratic term may be ignored in the IR:

\[ Z = \int D\phi D\sigma \ e^{-\int d^d x \left( \frac{1}{2} (\partial \phi^i)^2 + \frac{1}{2\sqrt{N}} \sigma \phi^i \phi^i \right)} \]

\[ = \int D\sigma \ e^{\frac{1}{8N} \int d^d x d^d y \sigma(x) \sigma(y) \langle \phi^i \phi^i(x) \phi^j \phi^j(y) \rangle_0 + \mathcal{O}(\sigma^3)} \]

• **Induced dynamics** for the auxiliary field endows it with the propagator

\[
\langle \sigma(p) \sigma(-p) \rangle = 2^{d+1} (4\pi)^{\frac{d-3}{2}} \Gamma \left( \frac{d-1}{2} \right) \sin \left( \frac{\pi d}{2} \right) (p^2)^{2-\frac{d}{2}} \equiv \tilde{C}_\sigma (p^2)^{2-\frac{d}{2}}
\]

\[
\langle \sigma(x) \sigma(y) \rangle = \frac{2^{d+2} \Gamma \left( \frac{d-1}{2} \right) \sin \left( \frac{\pi d}{2} \right)}{\pi^{\frac{3}{2}} \Gamma \left( \frac{d}{2} - 2 \right)} \frac{1}{|x-y|^4} \equiv \frac{C_\sigma}{|x-y|^4}
\]
• The $1/N$ corrections to operator dimensions are calculated using this induced propagator. For example,

$$\Delta_\phi = \frac{d}{2} - 1 + \frac{1}{N} \eta_1 + \frac{1}{N^2} \eta_2 + \ldots$$

• For the leading correction need

$$\frac{1}{N} \int \frac{d^d q}{(2\pi)^d} \frac{1}{(p-q)^2} \frac{\tilde{C}_\sigma}{(q^2)^{\frac{d}{2}-2+\delta}}$$

• $\delta$ is the regulator later sent to 0.

$$\eta_1 = \frac{\tilde{C}_\sigma (d-4)}{(4\pi)^{\frac{d}{2}} d \Gamma \left( \frac{d}{2} \right)} = \frac{2^{d-3} (d-4) \Gamma \left( \frac{d-1}{2} \right) \sin \left( \frac{\pi d}{2} \right)}{\pi^{\frac{3}{2}} \Gamma \left( \frac{d}{2} + 1 \right)}$$
• When the leading correction is negative, the large N theory is non-unitary.
• It is positive not only for $2<d<4$, but also for $4<d<6$.

• The 2-point function coefficient $C_\sigma$ is similar
Towards Interacting 5-d O(N) Model

- Scalar large N model with $\frac{\lambda}{4} (\phi^i \phi^i)^2$ interaction has a good UV fixed point for $4 < d < 6$. Parisi

- In $4 + \epsilon$ dimensions

$$\beta_{\lambda} = \epsilon \lambda + \frac{N + 8}{8 \pi^2} \lambda^2 + \ldots$$

- So, the UV fixed point is at a negative coupling

$$\lambda_* = -\frac{8 \pi^2}{N + 8} \epsilon + O(\epsilon^2)$$

- At large N, conjectured to be dual to Vasiliev theory in AdS$_6$ with $\Delta_-$ boundary condition on the bulk scalar. Giombi, IK, Safdi

- Check of 5-dimensional F-theorem

$$F_{UV}^{(1)} - F_{IR}^{(1)} = -\frac{3 \zeta(5) + \pi^2 \zeta(3)}{96 \pi^4} \approx -0.0016$$

$$-F = \log Z_{S^5}$$
Perturbative IR Fixed Points

- Work in \( d = 6 - \epsilon \) with O(N) symmetric cubic scalar theory

\[
\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^i)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{g_1}{2} \sigma (\phi^i \phi^i) + \frac{g_2}{6} \sigma^3
\]

- The beta functions Fei, Giombi, IK

\[
\begin{align*}
\beta_1 &= -\frac{\epsilon g_1}{2} + \frac{(N - 8)g_1^3 - 12g_1^2g_2 + g_1g_2^2}{12(4\pi)^3} \\
\beta_2 &= -\frac{\epsilon g_2}{2} + \frac{-4Ng_1^3 + Ng_1^2g_2 - 3g_2^3}{4(4\pi)^3}
\end{align*}
\]

- For large \( N \), the IR stable fixed point is at real couplings

\[
g_{1*} = \sqrt{\frac{6\epsilon (4\pi)^3}{N}} \\
g_{2*} = 6g_{1*}
\]
RG Flows

• Here is the flow pattern for $N=2000$

• The IR stable fixed points go off to complex couplings for $N < 1039$. Large N expansion breaks down very early!
• The dimension of sigma is
\[ \Delta_\sigma = 2 - \frac{\epsilon}{2} + \frac{Ng_1^2 + g_2^2}{12(4\pi)^3} \]

• At the IR fixed point this is
\[ 2 + 40\frac{\epsilon}{N} \]

• Agrees with the large N result for the O(N) model in d dimensions:

Petkou (1995)

\[ 2 + \frac{4}{N} \frac{\Gamma(d)}{\Gamma(d/2 - 1)\Gamma(1 - d/2)\Gamma(d/2)\Gamma(d/2 + 1)} \]

• For N=0, the fixed point at imaginary coupling may lead to a description of the Lee-Yang edge singularity in the Ising model. Michael Fisher (1978)

• For N=0, \( \Delta_\sigma \) is below the unitarity bound \( 2 - \frac{\epsilon}{2} \)

• For N>1039, the fixed point at real couplings is consistent with unitarity in \( d = 6 - \epsilon \)
The beta functions are found to be

\[
\beta_1 = -\frac{\epsilon}{2}g_1 + \frac{1}{12(4\pi)^3}g_1 \left( (N - 8)g_1^2 - 12g_1g_2 + g_2^2 \right)
- \frac{1}{432(4\pi)^6}g_1 \left( (536 + 86N)g_1^4 + 12(30 - 11N)g_1^3g_2 + (628 + 11N)g_1^2g_2^2 + 24g_1g_2^3 - 13g_2^4 \right)
+ \frac{1}{62208(4\pi)^9}g_1 \left( g_2^6(5195 - 2592\zeta(3)) + 12g_1g_2^5(-2801 + 2592\zeta(3)) \right)
- 8g_1^2g_2^4(1245 + 119N + 7776\zeta(3)) + g_1^4g_2^2(-358480 + 53990N - 3N^2 - 2592(-16 + 5N)\zeta(3))
+ 36g_1^5g_2(-500 - 3464N + N^2 + 864(5N - 6)\zeta(3))
- 2g_1^6(125680 - 20344N + 1831N^2 + 2592(25N + 4)\zeta(3)) + 48g_1^3g_2^3(95N - 3(679 + 864\zeta(3))) \right)
\]

\[
\beta_2 = -\frac{\epsilon}{2}g_2 + \frac{1}{4(4\pi)^3} \left( -4Ng_1^3 + Ng_1^2g_2 - 3g_2^3 \right)
+ \frac{1}{144(4\pi)^6} \left( -24Ng_1^5 - 322Ng_1^4g_2 - 60Ng_1^3g_2^2 + 31Ng_1^2g_2^3 - 125g_2^5 \right)
+ \frac{1}{20736(4\pi)^9} \left( -48N(713 + 577N)g_1^7 + 6272Ng_1^2g_2^5 + 48Ng_1^3g_2^4(181 + 432\zeta(3)) \right)
- 5g_2^7(6617 + 2592\zeta(3)) - 24Ng_1^5g_2^2(1054 + 471N + 2592\zeta(3))
+ 2Ng_1^6g_2(19237N - 8(3713 + 324\zeta(3))) + 3Ng_1^4g_2^3(263N - 6(7105 + 2448\zeta(3))) \right)
\]
The epsilon expansions of scaling dimensions agree in detail with the large N expansion at the UV fixed point of the quartic O(N) model:

\[
\Delta_\phi = \frac{d}{2} - 1 + \gamma_\phi \\
= 2 - \frac{\epsilon}{2} + \left( \frac{1}{N} + \frac{44}{N^2} + \frac{1936}{N^3} + \ldots \right) \epsilon + \left( -\frac{11}{12N} - \frac{835}{6N^2} - \frac{16352}{N^3} + \ldots \right) \epsilon^2 \\
+ \left( -\frac{13}{144N} + \frac{6865}{72N^2} + \frac{54367/2}{N^3} - 3672\zeta(3) + \ldots \right) \epsilon^3,
\]

\[
\Delta_\sigma = \frac{d}{2} - 1 + \gamma_\sigma \\
= 2 + \left( \frac{40}{N} + \frac{6800}{N^2} + \ldots \right) \epsilon + \left( -\frac{104}{3N} - \frac{34190}{3N^2} + \ldots \right) \epsilon^2 \\
+ \left( -\frac{22}{9N} + \frac{47695/18}{N^2} - 2808\zeta(3) + \ldots \right) \epsilon^3.
\]
Critical N

• What is the critical value of $N$ below which the perturbatively unitary fixed point disappears?

• Need to find the solution of

$$\beta_1 = 0, \quad \beta_2 = 0,$$
$$\frac{\partial \beta_1}{\partial g_1} \frac{\partial g_1}{\partial \beta_1} = \frac{\partial \beta_2}{\partial g_2} \frac{\partial g_2}{\partial \beta_2}$$

• The three loop calculation gives

$$N_{\text{crit}} = 1038.266 - 609.840\epsilon - 364.173\epsilon^2 + \mathcal{O}(\epsilon^3)$$
(Meta) Stability

- Since the UV lagrangian is cubic, does the theory make sense non-perturbatively?
- When the CFT is studied on $S^d$ or $R \times S^{d-1}$ the conformal coupling of scalar fields to curvature renders the perturbative vacuum meta-stable. In 6-$\varepsilon$ dimensions, scaling dimensions have imaginary parts of order $\exp(-\frac{A N}{\varepsilon})$
- This suggests that the dual Vasiliev theory is metastable, but only for the $\Delta$ boundary conditions corresponding to the interacting $O(N)$ model.
Higher-Spin dS/CFT

• To construct non-unitary CFTs dual to higher spin theory in de Sitter space, replace the commuting scalar fields by anti-commuting ones. Anninos, Hartman, Strominger

• The conjectured dual to minimal Vasiliev theory in dS\textsubscript{4} is the interacting Sp(N) model introduced earlier LeClair, Neubert

\[
S = \int d^3x \left( \frac{1}{2} \Omega_{ij} \partial_{\mu} \chi^i \partial^{\mu} \chi^j + \frac{1}{4} \Omega_{ij} \chi^i \chi^j \right)
\]
• In \( d>4 \) this quartic theory has a UV fixed point at large \( N \).
• Consider instead the cubic \( \text{Sp}(N) \) invariant theory, which is weakly coupled in \( 6-\varepsilon \) dimensions.

\[
S = \int d^dx \left( \frac{1}{2} \Omega_{ij} \partial_\mu \chi^i \partial^\mu \chi^j + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} g_1 \Omega_{ij} \chi^i \chi^j \sigma + \frac{1}{6} g_2 \sigma^3 \right)
\]
• The beta functions are related to those of the \( \text{O}(N) \) theory via \( N \rightarrow -N \)
• For \( \text{Sp}(N) \) there are IR stable fixed points at imaginary couplings for all positive even \( N \).
Symmetry Enhancement for N=2

• The N=2 model may be written as

\[ S = \int d^d x \left( \partial_\mu \theta \partial^\mu \bar{\theta} + \frac{1}{2} (\partial_\mu \sigma)^2 + g_1 \sigma \theta \bar{\theta} + \frac{1}{6} g_2 \sigma^3 \right) \]

• At the fixed point

\[ g_2^* = 2 g_1^*, \quad g_1^* = i \sqrt{\frac{(4\pi)^3 \epsilon}{5}} \left( 1 + \frac{67}{180} \epsilon + O(\epsilon^2) \right) \]

• There is symmetry enhancement to the supergroup Osp(1|2)

\[ \delta \theta = \sigma \alpha, \quad \delta \bar{\theta} = \sigma \bar{\alpha}, \quad \delta \sigma = -\alpha \bar{\theta} + \bar{\alpha} \theta \]
• Defining \[ g_1 = i \sqrt{\frac{(4\pi)^3 \epsilon}{5}} x, \quad g_2 = i \sqrt{\frac{(4\pi)^3 \epsilon}{5}} y \]

• The scaling dimensions of commuting and anti-commuting scalars are equal

\[ \Delta_\sigma = \Delta_\theta = 2 - \frac{8}{15} \epsilon - \frac{7}{450} \epsilon^2 - \frac{269 - 702 \zeta(3)}{33750} \epsilon^3 \]
Connection with the Potts Model

• (n+1) state Potts model can be described in $6-\varepsilon$ dimensions by a cubic field theory of n scalar fields Zia, Wallace

$$ S = \int d^d x \left( \partial_\mu \phi^i \partial^\mu \phi^i + \frac{1}{6} g d_{ijk} \phi^i \phi^j \phi^k \right) $$

$$ d_{ijk} = \sum_{\alpha=1}^{n+1} e^i_\alpha e^j_\alpha e^k_\alpha $$

• The vectors $e^i_\alpha$ describe the vertices of n-dimensional generalization of tetrahedron.
• The critical exponent

\[ \eta = \frac{1 - n}{3(-7 + 3n)} e - \frac{(-1 + n)(206 - 171n + 43n^2)}{27(-7 + 3n)^3} \epsilon^2 \]

\[ -\frac{1 + n}{972(-7 + 3n)^5} \left( -187238 + 300903n - 210179n^2 + 68025n^3 - 8375n^4 
+ (580608 - 829440n + 466560n^2 - 129600n^3 + 15552n^4)\zeta(3) \right) \epsilon^3 \]

• Determines the scaling dimension via

\[ \Delta_\theta = \frac{d}{2} - 1 + \frac{n}{2} \]

• Taking the limit \( n \to -1 \) corresponding to zero-state Potts model, we find agreement with the epsilon expansion for the Osp(1|2) model!
• The appearance of the Osp(1|2) symmetry in the zero-state Potts model was noted earlier in a different way: using the 2-d OSp(1|2) sigma model. Caracciolo, Jacobsen, Saleur, Sokal, Sportiello

• The zero-state Potts model can be simulated on a lattice using the spanning forests, and Monte Carlo results for scaling exponents are available in d=3,4,5 where the model has second order phase transitions. Deng, Garoni, Sokal

• Perhaps the spanning forests on a lattice are related to strongly coupled higher-spin quantum gravity in de Sitter space?
More double-trace

- Can consider double-trace perturbations

\[ S = S_0 + \frac{\lambda_0}{2} \int d^3x \sqrt{g} J_{\mu_1\mu_2...\mu_s}(x) J^{\mu_1\mu_2...\mu_s}(x) \]

- \( J \) is a spin-s single trace operator of dimension \( \Delta \). Unitarity bound: \( \Delta^{(s)} \geq s + 1 \)

- Generalized Hubbard-Stratonovich transformation

\[ S = S_0 - \int d^3 x \sqrt{g(x)} \left[ h_{\mu_1...\mu_s}(x) J^{\mu_1...\mu_s}(x) + \frac{1}{2\lambda_0} h_{\mu_1...\mu_s} h^{\mu_1...\mu_s} \right] \]
Conserved Currents

• When $J_{\mu_1\mu_2...\mu_s}$ is conserved then $\Delta = s + 1$

• The auxiliary field becomes a spin $s$ gauge field with gauge transformation

$$\delta h_{\mu_1\mu_2...\mu_s} = (\mathcal{O}_g v)_{\mu_1\mu_2...\mu_s}$$

$$(\mathcal{O}_g v)_{\mu_1\mu_2...\mu_s} = \nabla(\mu_1 v_{\mu_2\mu_3...\mu_s}) - \frac{s - 1}{d + 2(s - 2)} g(\mu_1\mu_2 \nabla^\nu v_{\mu_3\mu_4...\mu_s})\nu$$

• For example, in $d$-dimensional QED the induced kinetic term is

$$F_{\mu\nu}(-\nabla^2)^{d/2-2} F^{\mu\nu}$$
Dual Analysis in AdS\(_{d+1}\)

- Fierz-Pauli equations for massive spin \(s\) fields
  \[
  (\nabla^2 + 2 - (s - 2)(d + s - 3) - m^2) h_{\mu_1...\mu_s} = 0 \\
  \nabla^\mu h_{\mu_2...\mu_s} = 0, \quad g^{\mu\nu} h_{\mu_3...\mu_s} = 0.
  \]

- Follow from minimal actions like the Proca action
  \[
  S = \int d^{d+1}x \sqrt{g} \left( \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu \right)
  \]

- Dimensions of single trace spin \(s\) operators
  \[
  \Delta_{\pm} = \frac{d}{2} \pm \nu, \quad \nu = \sqrt{m^2 + \left(\frac{d}{2} + s - 2\right)^2}
  \]
• Boundary behavior in $H^{d+1}$

$$ds^2 = \frac{dz^2}{z^2} + \sum_{i=1}^{d} dx_i^2$$

$$h_{i_1\cdots i_s} \sim z^{\Delta-s}$$

• The UV CFT is associated with the more singular boundary behavior

• Change in the free energy on $S^d$ Gubser, Mitra

$$\delta F^{(s)}_\Delta = F^{(s)}_\Delta - F^{(s)}_\Delta = \frac{1}{2} \left[ \text{tr}^{(s)}_\Delta \log(-\nabla^2 + m^2) - \text{tr}^{(s)}_+ \log(-\nabla^2 + m^2) \right]$$

$$\partial_\Delta \delta F^{(s)}_\Delta = (2\Delta - d) \frac{\partial \delta F^{(s)}_\Delta}{\partial m^2} = \frac{2\Delta - d}{2} \int \text{vol}_{H^{d+1}} \left( \text{Tr} G^{(s)}_\Delta(x,x) - \text{Tr} G^{(s)}_\Delta(x,x) \right)$$
Massless spin $s$ in AdS

- The one-loop partition function is Gibbons, Hawking, Perry; Gaberdiel, Gopakumar, Saha; Gaberdiel, Grumiller, Vassilevich; ...

$$Z_s = \frac{\left[\det^{STT}_{s-1} \left(-\nabla^2 + (s-1)(d+s-2)\right)\right]^{\frac{1}{2}}}{\left[\det^STT_s \left(-\nabla^2 + (s-2)(d+s-3) - 2\right)\right]^{\frac{1}{2}}}$$

- Roughly, the numerator is due to spin $s-1$ ghosts.

- The ghost boundary conditions are correlated with those of spin $s$ fields

$$\delta h_{\mu_1...\mu_s} = \nabla_{(\mu_1} \xi_{\mu_2...\mu_s)}$$

$$\xi_{i_1...i_{s-1}}(z, x_i) \sim z^{\delta_{+}} c_{i_1...i_{s-1}}(x_i), \quad \delta_{+} = d, \quad \delta_{-} = 2 - 2s$$
Even d and Weyl Anomalies

- In even dimensions $d$, the sphere free energy may be used to calculate the $a$-coefficient of the Weyl anomaly, i.e. the coefficient of the Euler density.
- The induced gauge theories have local actions which are classically Weyl invariant.
- In $d=4$, for spin 1 it is the Maxwell theory; for spin 2 it is the conformal gravity with Weyl-squared action.
- Quadratic actions have been generalized to $s>2$ by Fradkin and Tseytlin. These conformal higher spin theories have higher derivative actions.
4-d Weyl Anomalies from AdS$_5$

- The 1-loop partition function in AdS$_5$ provides a remarkably simple approach to the Weyl anomaly calculations: 

\[ a_s = a_s^{\text{phys}} - a_{s-1}^{\text{ghost}} \]

\[ a_s = \frac{s^2}{180} (1 + s)^2 [3 + 14s(1 + s)] \]

- For $s=1$ this reproduces $31/45$ of the Maxwell field; for $s=2$ it reproduces $87/5$ Fradkin, Tseytlin. For higher $s$ these are predictions of AdS/CFT.

- Confirmed by a direct 4-d calculation. Tseytlin
6-d Weyl Anomalies from AdS$_7$

- Similarly, one-loop partition functions in AdS7 give Giombi, IK, Safdi; Tseytlin

\[- \frac{(\gamma_6 - \frac{1}{4})^2}{4838400} (25 - 884\gamma_6 + 2288\gamma_6^2 - 704\gamma_6^3) \, , \quad \gamma_6 = \left( s + \frac{3}{2} \right)^2\]

- For $s=0$, this gives $-1/756$, which is the usual 6-d conformally coupled scalar Weyl anomaly.
- For $s=1$, it gives $55/84$ corresponding to the conformal 4-derivative Maxwell theory Smilga ...

$$F_{\mu\nu}(-\nabla^2) F^{\mu\nu}$$
Conclusions

• Found a new description of the meta-stable fixed points of the scalar O(N) model in 4<d<6 valid for sufficiently large N.
• Suggested existence of an instanton in the dual Vasiliev theory for ‘minus’ boundary conditions.
• Extensions to Sp(N) invariant theories including N anti-commuting scalar fields, which may be dual to HS theories in de Sitter spaces.
• The N=2 case exhibits symmetry enhancement to OSp(1|2) and is related to the zero-state Potts model.
• Weyl anomalies for CHS fields.